

Special Report 10

**RADAR REFLECTION COEFFICIENTS
FROM A PLASMA GRADIENT WITH COLLISIONS (U)**

By: WALTER G. CHESNUT

Prepared for:

DEFENSE ATOMIC SUPPORT AGENCY
WASHINGTON, D.C. 20305

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STANFORD RESEARCH INSTITUTE

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By: WALTER G. CHESNUT

SRI Project 4417

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ABSTRACT

This report presents numerical results in the form of graphs of the power reflection coefficient for electromagnetic signals normally incident upon a plasma gradient. An electron collision frequency that is independent of position in the plasma interface is assumed. A "slide rule" is provided to enable quick determination of the magnitude of the reflection coefficient for "very overdense, exponential plasmas" in the lower atmosphere (to 70 km) for frequencies from 30 MHz to 10,000 MHz. For plasmas that only reach a finite density, a complete set of graphs is given presenting power-reflection coefficients for normalized plasma parameters. Reflectivities are plotted if greater than 10^{-20} . Normalized collision frequencies (Z) from 10^{-3} to 10^{+3} are covered. Normalized plasma to RF frequency ratios inside the plasma cover the range from 0.01 to 100 ($10^{-4} \leq X \leq 10^{+4}$). Characteristic distances over which the plasma density exponentiates in the gradient region vary from 10^{-3} wavelengths to 10^{+3} wavelengths. Simple aids to the reader are provided so that he may easily determine his parameters for use with the reflectivity curves, should his plasma be of atmospheric constituents.

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The author specially appreciates the careful computer programming provided by Mrs. Lorie McNiel. Mrs. McNiel was responsible for the forming of the computer drawn graphs so that optimum choices of plasma frequencies would be used in order to avoid large gaps or dense crowding on the graphs. She was also responsible for the "log-wise" stepping of the abscissa values in order to obtain smoother--and thereby--more useful plots.

Mr. Dennis Allison has checked the author's mathematical steps and procedures for accuracy.

1 INTRODUCTION

This report presents numerical results for the power-reflection coefficient for electromagnetic waves normally incident upon an interface between vacuum and a plasma. Specifically, numerical results are presented for situations in which the plasma density varies gradually over a wavelength of the electromagnetic wave. Collisions between the plasma electrons and a neutral background gas are included.

Two types of plasma density profiles are considered. In the first, the plasma density is allowed to exponentiate to infinite density. For many applications, this approximation is adequate because in practice wave attenuation prevents the incident signal from actually interacting significantly with the very high electron densities existing at great penetration depths. Therefore the details of electron density very deep within the medium do not matter. The solution for the power reflectivity in this case is a very simple analytic expression that admits to simple evaluation. The power-reflection coefficient in a gradient of this sort is presented in the form of a "slide rule" to allow the reader to choose his own exponentiating distance. The reflection coefficient is presented as a function of altitude in the earth's atmosphere for frequencies from 30 MHz to 10,000 MHz.

The second density profile that is considered is one form of the "Epstein" profile--a profile that permits, in principle, a very wide latitude in choice of plasma layer shapes. In practice, a numerical solution requires evaluation of rather unfamiliar analytic functions. For our example, the function is the gamma function of complex argument. The plasma profile considered here varies monotonically from vacuum to a finite electron density. The electron collision frequency is assumed to be constant throughout the plasma (electron-neutral collisions are assumed to dominate). For very high electron densities deep within the plasma region, this particular profile appears to be nearly the same as the exponential profile considered in the first part.

The determination of the absolute magnitude of gamma function of complex arguments used in evaluating the reflectivity of the Epstein layer requires evaluation of a continued product. A computer test has revealed that for many cases of interest in this work, 10,000 terms of the continued product have not converged the reflectivity to within 20 percent of the proper result. We have therefore developed a technique of analytic approximation to the continued product that allows very rapid convergence to one-percent accuracy. Details are given in the Appendix. The technique may be of interest to others faced with evaluation of similar analytical functions. One form of our analytic technique enables slide-rule calculation of power reflectivity to an accuracy of 10 percent should the reflectivity be greater than 10^{-20} . Therefore the reader does have at his disposal a rather simple, analytical technique that he may apply for situations of particular interest not covered by our normalized graphs.

Section 2 of the report presents background information concerning plasma properties. Section 2 also provides disclaimers, reservations, admonitions, and qualifications reflecting the author's own practical point of view regarding reflectivity calculations. Section 3 describes the reflectivity from an exponential plasma gradient to infinite electron density. Section 4 presents the results of calculations for gradients to finite electron density. The mathematical developments are contained in the Appendix.

This report will show that coherent reflection from an interface between a plasma and a vacuum can be extremely reduced if the interface is gradual. The coherent reflection can become very small for some plasmas that actually have very high electron densities. In this circumstance, it seems likely that energy can be returned to the signal source by any one of many other kinds of reflection mechanisms. In order to decide if this may be the case, the author suggests that if the "Fresnel" term^{*} for scattering from the plasma is large (say, greater

^{*}We use this terminology to mean the reflectivity that would result if the plasma gradient were very short compared with the wavelength of the electromagnetic wave. Section 4 and the Appendix further clarify this term.

than 10^{-3} to 10^{-4}), yet application of the gradient degrades this to less than, say, 10^{-10} , then some other scattering mechanism is liable to be more important than the coherent reflection from the plasma gradient. An example would be scattering from turbulence or plasma waves that produce density fluctuations within the plasma gradient. This scattering is best calculated using Born approximation and a stochastic description of the dielectric fluctuations. Furthermore, under these same conditions of large Fresnel term, incoherent backscatter from free electrons will begin to be as intense as coherent effects due to the mean plasma structure at power reflectivity values below the order of 10^{-20} . For this reason, computer plotted data presented here do not extend below 10^{-20} .

2 BACKGROUND

The complex dielectric constant of a plasma is described approximately by the Appleton-Hartree formula.^{1*} Under the assumption that magnetic field effects are small, this dielectric constant is given by

$$\epsilon = 1 - \frac{X}{1 - iZ} \quad (1)$$

where

$$X = (f_p/f)^2$$

f_p = Plasma frequency in Hertz

f = Frequency of the electromagnetic wave in Hertz

$$Z = \nu/\omega = f_c/f$$

ν = Collision frequency in radians/second

$$\omega = 2\pi f$$

f_c = Collision frequency in Hertz.

The plasma frequency is related to electron density through

$$f_p = 8.97 \cdot 10^3 \sqrt{n_e} \text{ Hertz} \quad (2)$$

where n_e = electron density in number per cubic centimeter.

In the earth's atmosphere, Shkarofsky² has shown that the quantities Z and X used in Eq. (1) are not quite as we have defined them. The energy dependence of the electron collision cross section with nitrogen is such that one must use effective values for X and Z that are related to our definitions in a complex way. Figures 1 and 2 present curves for relating our definition of X and Z to X_{eff} and Z_{eff} to be used in calculations. For the data of Sec. 3, we have not made this correction,

*References are listed at the end of this report.

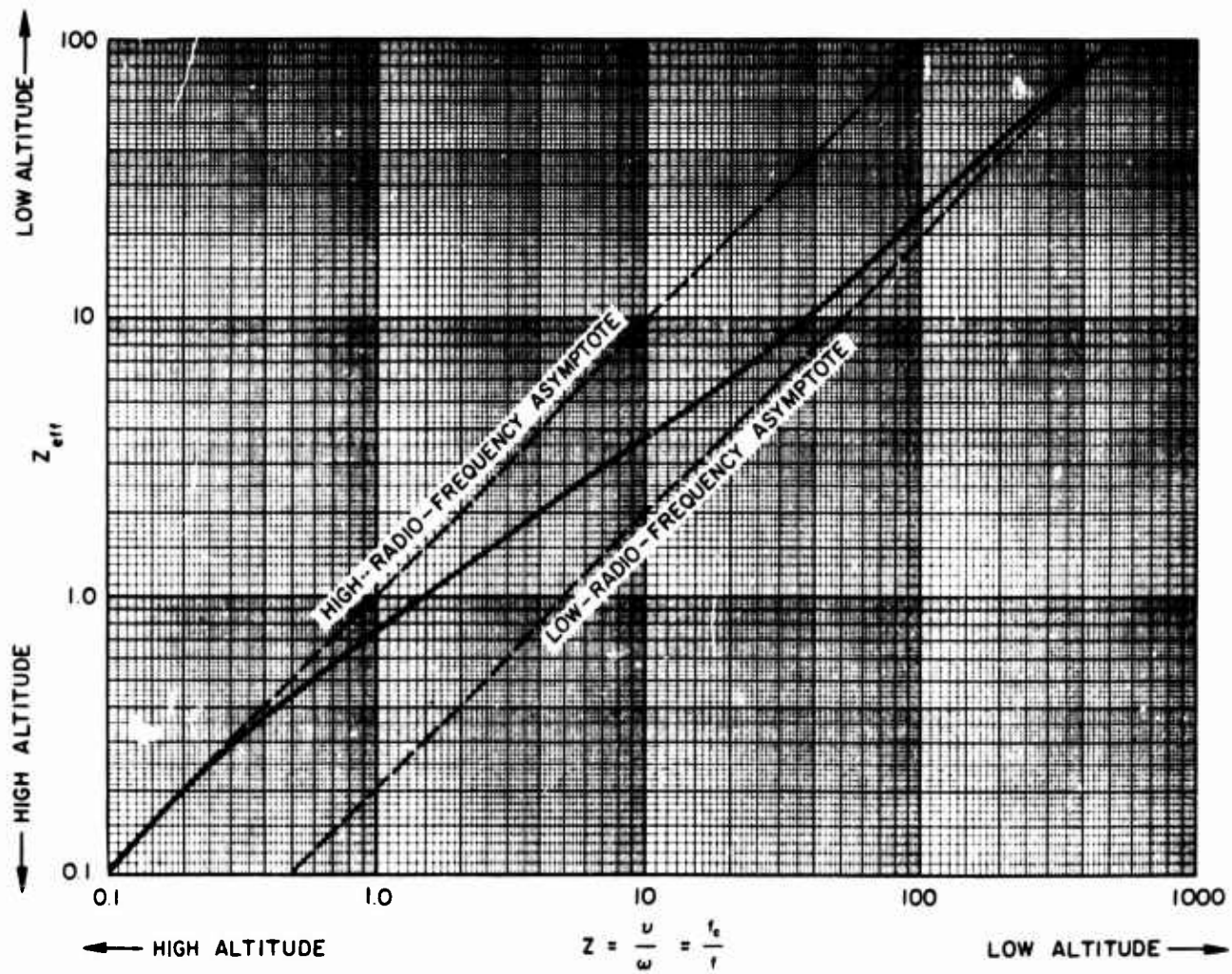


FIG. 1 CORRECTING Z TO Z_{eff} FOR USE IN APPLETON-HARTREE FORMULA-PROCEDURE AFTER SHKAROFKY (Ref. 2). Air Slightly Ionized.

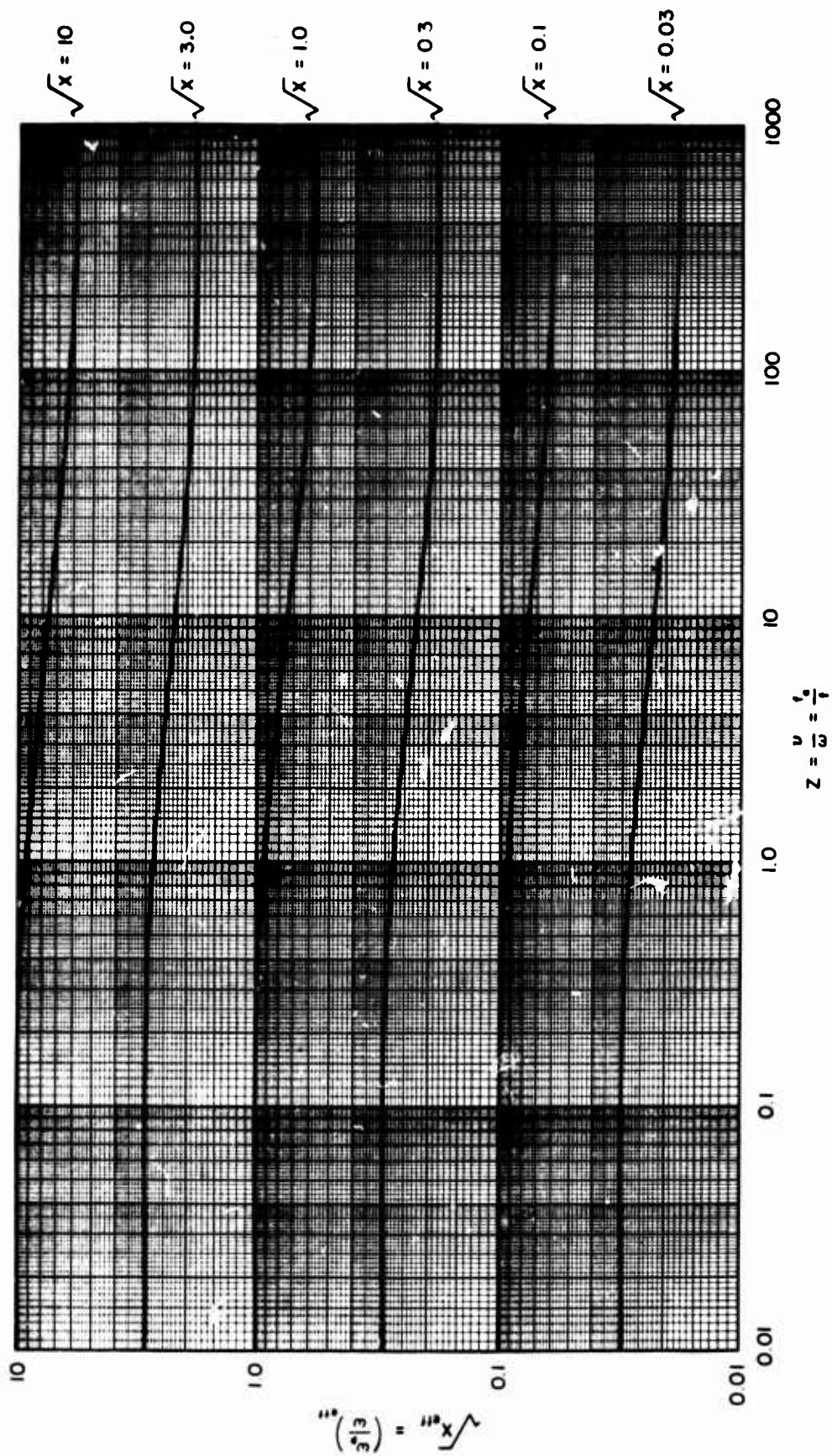


FIG. 2 $\omega_p/\omega = \sqrt{x}$ - CORRECTED FOR USE IN APPLETON-HARTREE FORMULA-PROCEDURE AFTER SHKAROFSKY (Ref. 2). Air Slightly Ionized.

though the effect, in a practical sense, is very minor. The reader may do as he wishes in using the data of Sec. 4.

To aid the reader we present in Figs. 3 and 4 our interpretation of the proper electron-neutral collision frequency as a function of altitude in the earth's atmosphere. In Fig. 3 the collision frequency is presented in radian measure. In Fig. 4 the collision frequency is presented in megahertz in order that the reader may more readily compute Z for his radar frequency. Equation (2) is plotted in Fig. 5; this curve gives the plasma frequency in megahertz versus electron density, which is needed to compute X .

The calculations to be presented assume that the electron collision frequency is constant throughout the plasma gradient. This situation pertains to a plasma in which the electron-neutral collision frequency is larger than other types of collision frequencies. The electron-ion collision frequency is proportional to electron density. Therefore, as the electron density increases, electron collisions with ions may become as large as the electron-neutral collision frequency. In the plasma density gradient, then, electron-ion collisions may begin to dominate the local propagation parameters. Our analytical results could be in error under these circumstances.

Figure 6 presents the electron-ion collision frequency versus electron density for three temperatures of air. Along the right-hand ordinate we have indicated altitudes from Fig. 3 at which the electron-neutral collision frequency attains the same value as the ordinate value. The reader must use care in employing the results to be presented later, particularly for high-frequency radars. That is, if the electron density in his plasma profile--for his air temperature--produces an electron collision frequency that equals or exceeds at some depth the electron neutral collision frequency, then he must attempt to determine whether penetration of his waves to that depth significantly affects the reflectivity to be expected. The reflectivity may be increased or decreased by this effect, according to details of the situation. We have not studied reflections from electron-density profiles in which collision frequency varies with position.

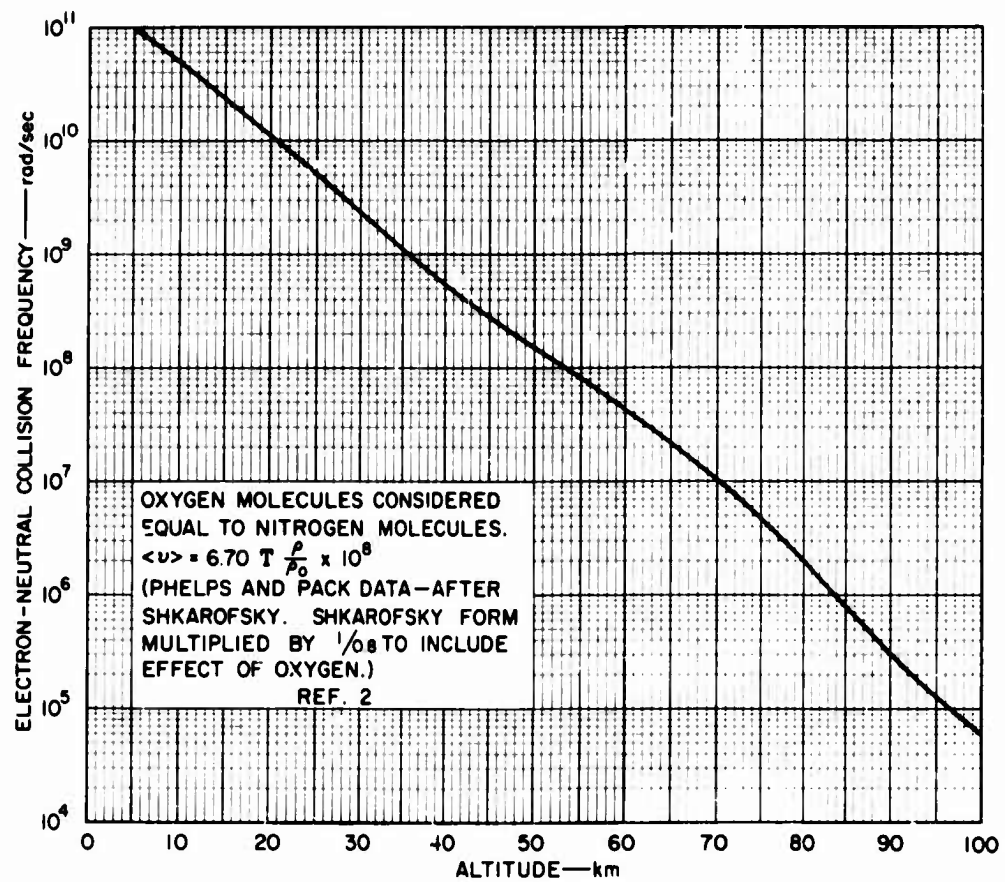


FIG. 3 ELECTRON-NEUTRAL COLLISIONAL FREQUENCY vs. ALTITUDE

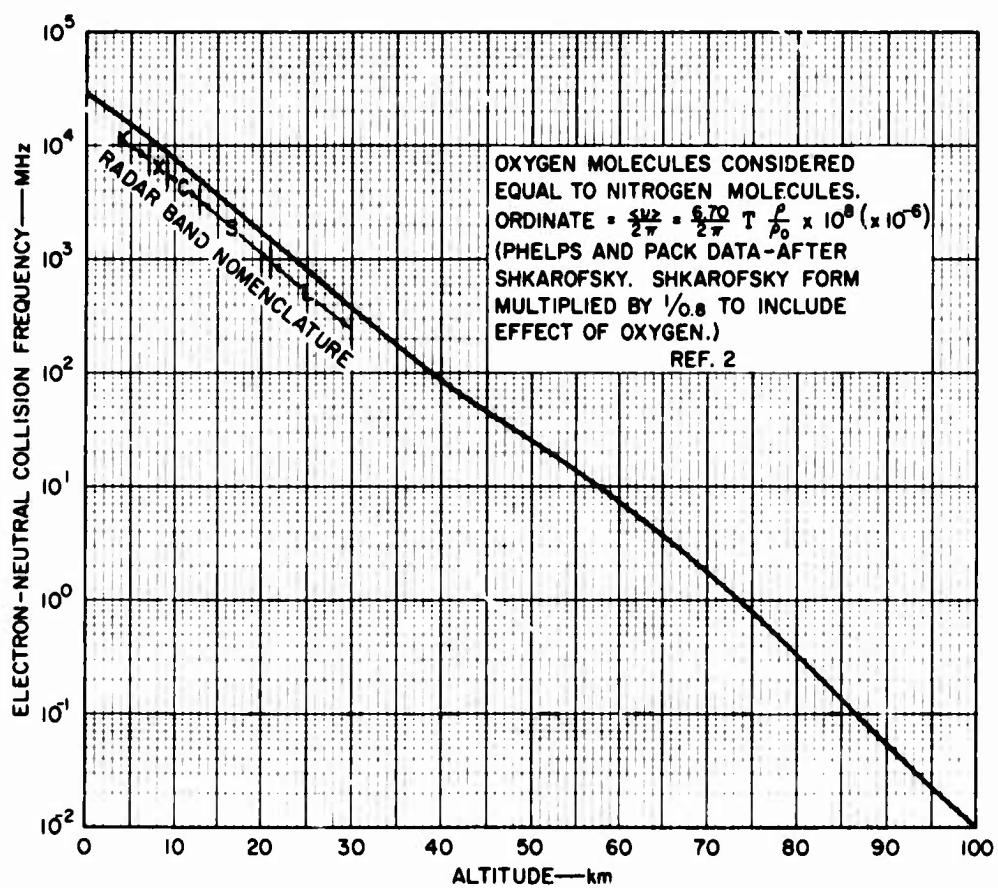


FIG. 4 ELECTRON-NEUTRAL COLLISION FREQUENCY EXPRESSED IN MEGAHERTZ

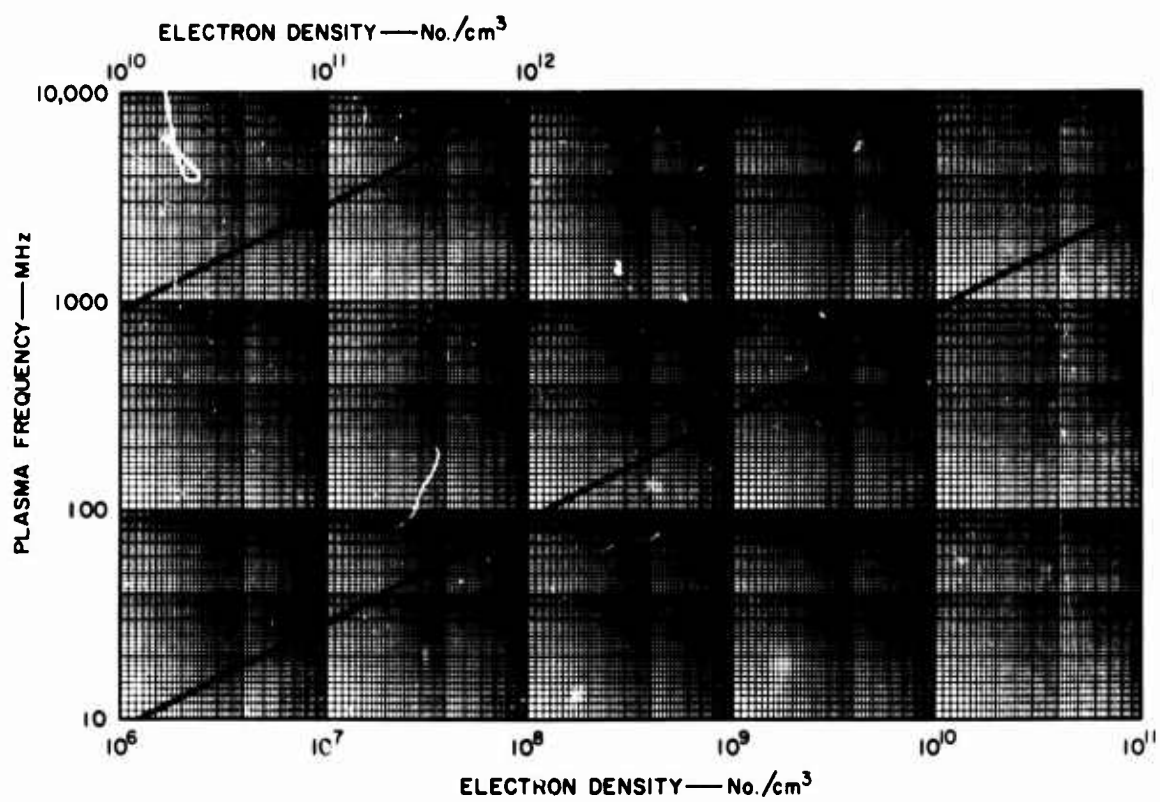


FIG. 5 PLASMA FREQUENCY vs. ELECTRON DENSITY

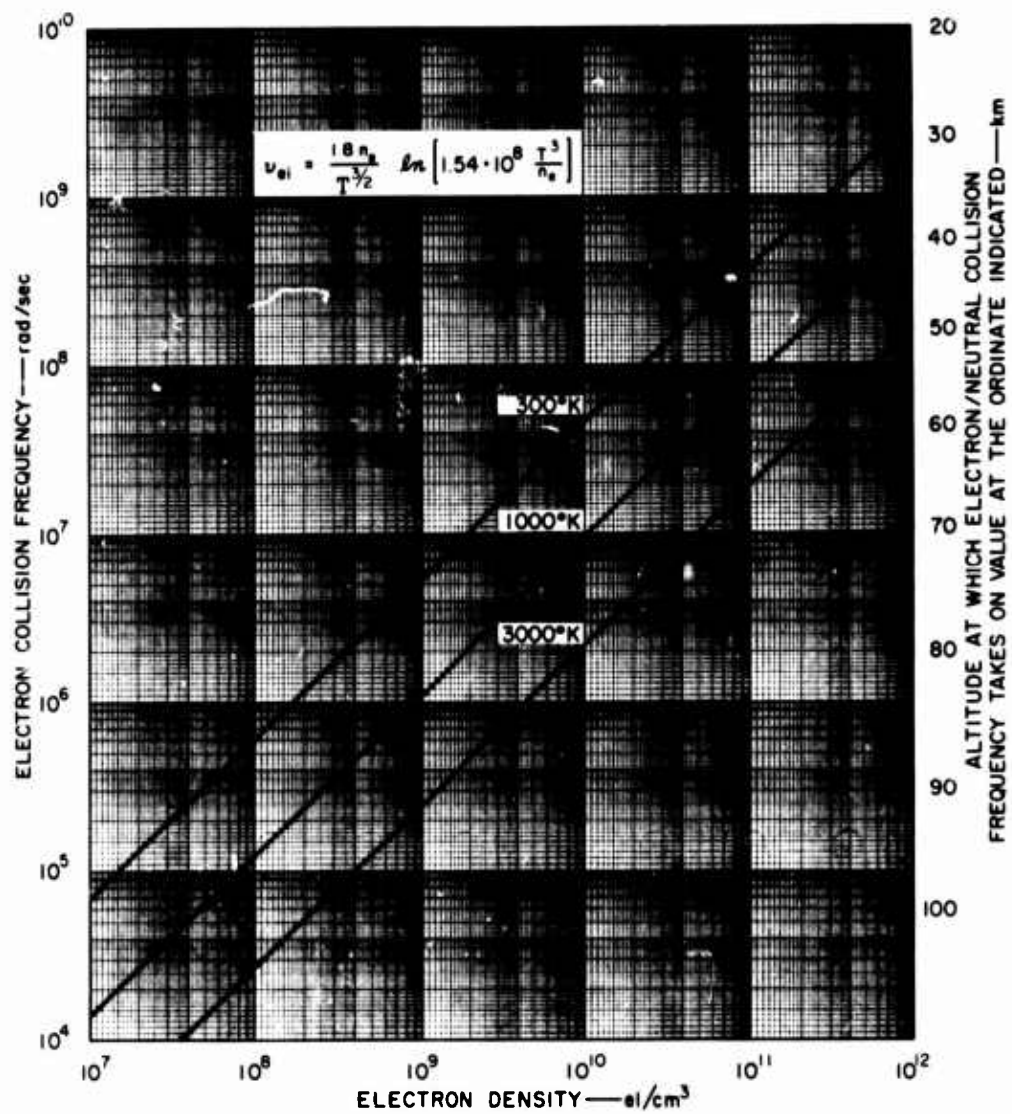


FIG. 6 ELECTRON-ION COLLISION FREQUENCY vs. ELECTRON DENSITY

The data to be presented were calculated for plasma gradients that are expressed in terms of simple, analytic functions. It is hardly expected that real plasma gradients will take on the forms that we use. We suggest therefore that our data be used primarily as a guide as to whether a coherent plasma interface reflection or some other reflection or scattering mechanism dominates a given measurement. We also suggest that if the plasma profile expected is very poorly approximated by our analytic expressions but the plasma frequency inside considerably exceeds the radar frequency, then the reader should match our analytic form to his plasma gradient at a position in the gradient where the radar wavelength in the plasma is longest compared with the plasma gradient. We can show that the radar wavelength in a plasma is

$$\lambda = \frac{\sqrt{2} \lambda_0}{\left\{ \left[\frac{(1-X)^2 + Z^2}{1+Z^2} \right]^2 + \frac{1+Z^2-X}{1+Z^2} \right\}^{\frac{1}{2}}} \quad (3)$$

where

λ = Radar wavelength in plasma

λ_0 = Radar wavelength in free space.

The wavelength takes on its greatest value at $X = 2$, if Z is finite. If Z is zero, then the power reflectivity is 1.0 regardless of plasma profile; here, profile details would be of concern only if the absolute phase of the return signal were of interest. The limit of λ with $Z \equiv 0$ is never met in practice, however, though some environments do approach this situation. Therefore, in a situation where the reader encounters a physical plasma profile very different from our analytical forms, he might match our analytic gradient at the position where $X = 2$ ($f_p = \sqrt{2} f$). On the other hand, our intuition suggests that penetration of the wave to the $X = 2$ depth may not be important, so that the gradient matching should be done at the $X = 1.0$ depth, particularly for very small values of Z . The choice for matching is left to the reader.

The main effect of electrons in the region where $X < 1.0$ is probably wave attenuation. Therefore, should the electron density profile in this

region be very different from our analytic forms, the reader may still wish to locally fit his physical gradient to our analytic gradient in the region from $X = 1$ to $X = 2$. He would then compute by WKB approximation the difference in absorption in the region where the profiles are very different, in order to obtain a more appropriate reflectivity.

Perhaps the most useful application of the data presented here would be to check results of computer programs that are designed to determine plasma reflectivities using a layer-by-layer numerical mock-up of the physical environment. One extreme danger of the mock-up procedure is that the amount of computer time required increases with the number of layers considered. Since the total number of layers that can be handled may be limited by computer capacity, the layer thicknesses tend to be picked too large. As a result, at each interface between layers (or between shells for a spherical geometry), the reflectivity may be larger than the total reflectivity of the medium. The person performing this kind of calculation then expects that, since he will be preserving phase, reflectivities from all the layers will tend to phase-cancel properly to leave the correct residual reflectivity. The difficulty with this procedure is that the reflectivity of one interface is likely to dominate, while all other interface reflectivities simply produce a sort of random walk away from this dominant interface value.

Our analytical results can be used to check such computer programs. The procedure would be to mock-up our analytic form, layer by layer, compute reflectivities, and then compare results. Good agreement would help develop confidence in the layer-by-layer mock-up procedure. Then the mock-up procedure could be applied to both plasma gradients and collision frequency variations that cannot be handled analytically.

3 REFLECTION FROM AN EXPONENTIAL PLASMA GRADIENT

The mathematical derivations of reflectivities used in this report were taken from Budden.³ Section 17.2 of Ref. 3 derives the reflectivity to be expected in a plasma gradient in which the electron density varies exponentially. The derivation assumes uniform collision frequency. We define

$$n_e(z) = n_o e^{z/z_o} \quad (4)$$

where

$n_e(z)$ = Electron density at position z

n_o = Electron density at position $z = 0$

z = Position in gradient ($-\infty < z < +\infty$)

z_o = Length over which the electron density changes by a factor of e . This value would be matched to the reader's problem.

In terms used in the previous section, Eq. (4) becomes

$$X \propto e^{z/z_o} \quad (5)$$

For normal incidence, Ref. 3 gives, for the power reflection coefficient [Eq. (17.17) of Ref. 3],

$$|R|^2 = \exp \left\{ - \frac{8\pi z_o}{\lambda} \tan^{-1} z \right\} \quad (6)$$

Figures 7 and 8 are to be used to evaluate Eq. (6). Figure 7 is a transparency to be used as an overlay on Fig. 8. The ordinant of Fig. 8 gives the power that 10 must be raised (always negative) in order to obtain the reflectivity.

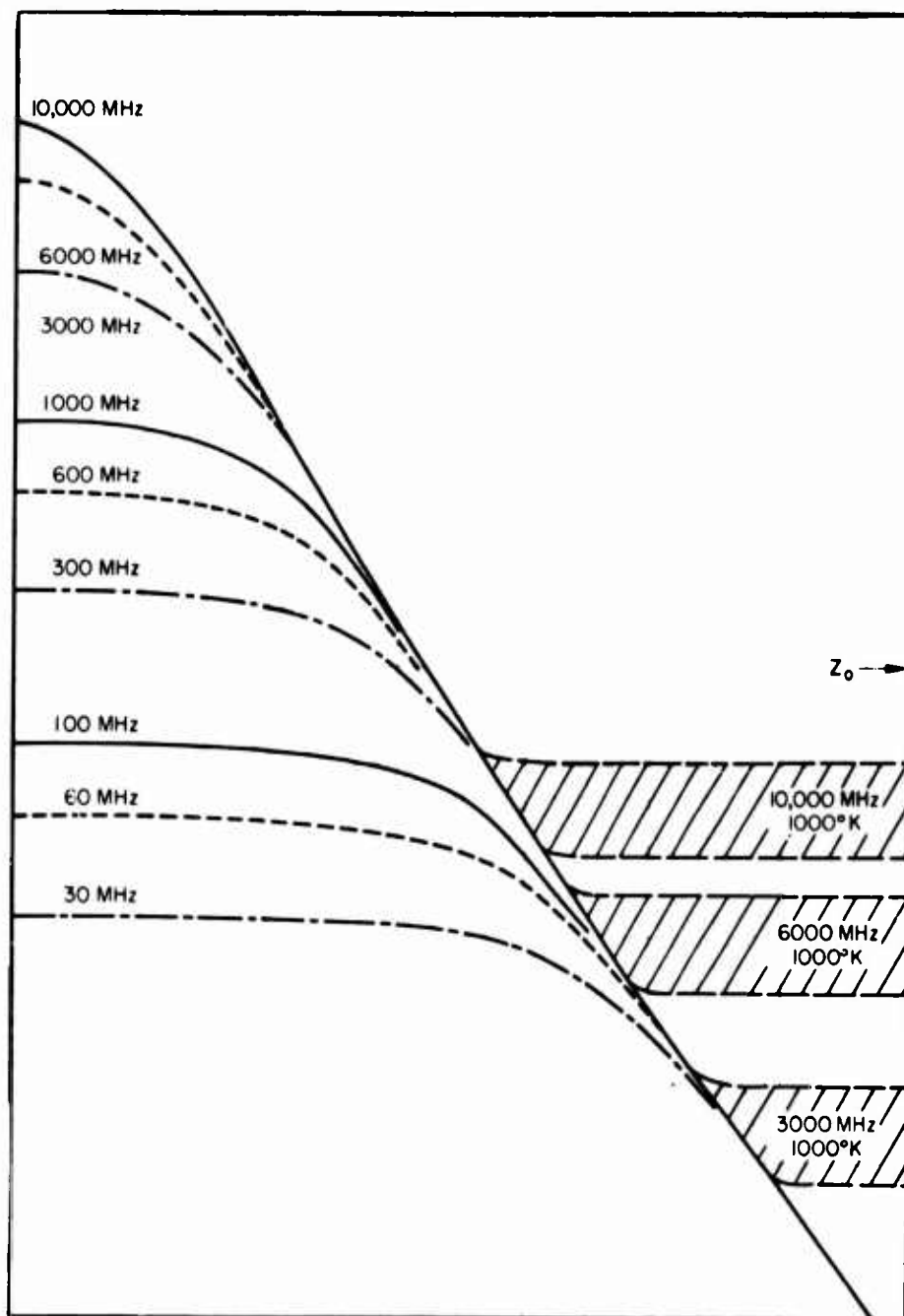


FIG. 7 OVERLAY FOR USE WITH FIG. 8 FOR CALCULATION OF REFLECTIVITY FROM EXPONENTIAL PLASMA

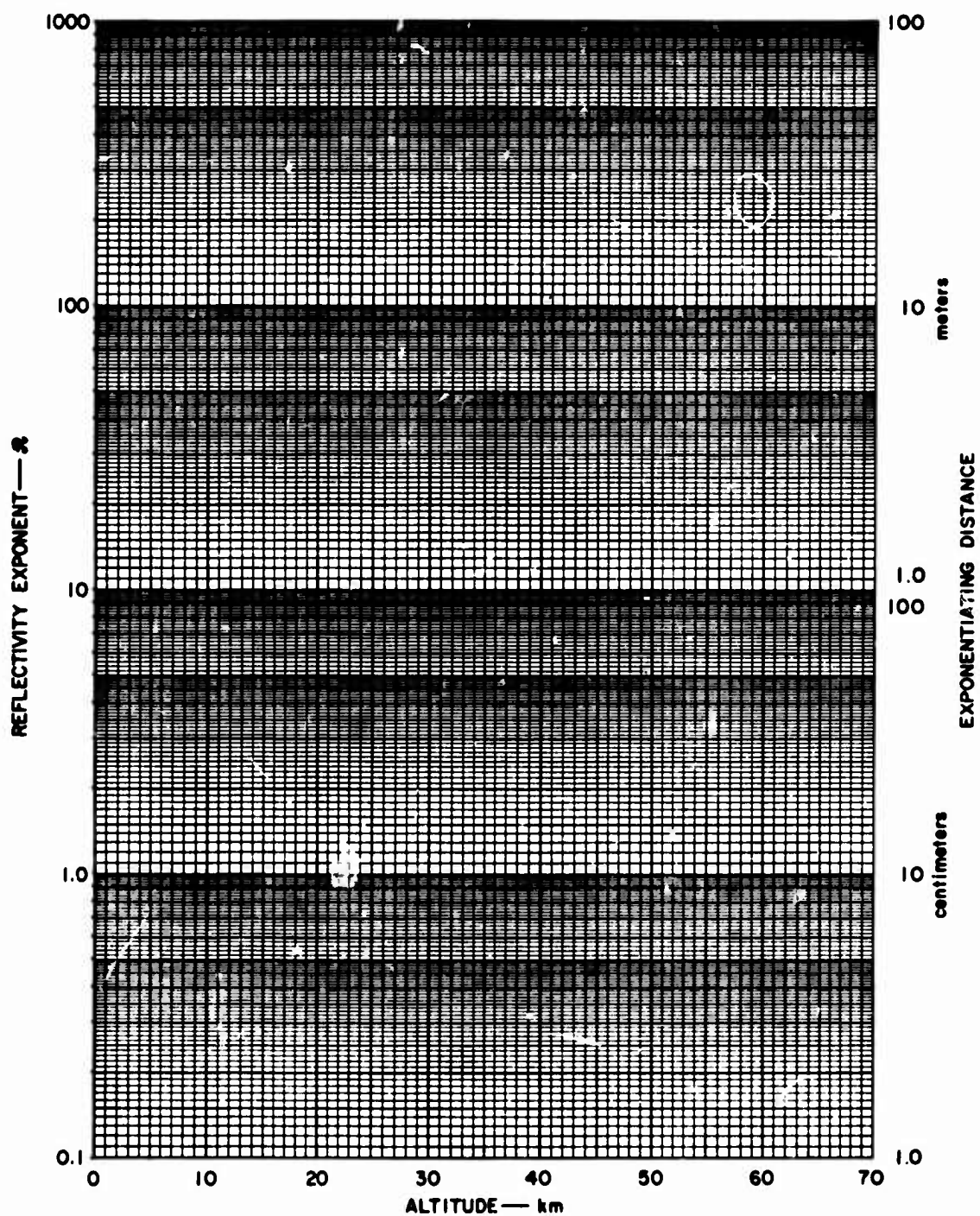


FIG. 8 SLIDE RULE FOR CALCULATING REFLECTION COEFFICIENT FOR EXPONENTIAL PLASMA GRADIENT

An acetate version of Fig. 7 (in pocket under back cover) is overlaid on Fig. 8 so that the arrow labeled z_0 is adjacent to the gradient distance of interest. For an exponential gradient of that value, the reader obtains the reflectivity exponent for any altitude or frequency of concern. As an example, let the exponentiating distance be one meter; then at an altitude of 10 km, the power-reflection coefficient for a 1000-MHz radar wave from a very overdense plasma is, approximately, 10^{-53} , clearly a reflectivity of little concern. On the other hand, a 100-MHz radar wave will experience a reflectivity value of $10^{-5.8}$, which may be of considerable concern.

At an altitude on the order of 40 km, at a plasma frequency of 10,000 MHz and a plasma temperature of 1000°K, the electron-neutral collision frequency and the electron-ion collision frequency are comparable. At higher altitudes than this the electron-ion collision frequency is greater than the electron-neutral collision frequency. Therefore, for 10,000 MHz, a plasma temperature of 1000°K, and altitudes above 40 km, the reflection coefficient is not given by Eq. (6). We have shown a shaded region that we believe probably contains the proper reflectivity value; at this writing we are not prepared to provide a precise value.

Similar shaded regions are given for 6000 and 3000 MHz, and 1000°K temperatures. For higher temperatures, the electron-ion collision frequency is lower, so that greater reflectivities will be obtained. We have chosen 1000°K for this discussion because air temperatures would generally have to be at least this high in order to maintain electron densities by radiation (or thermally) that are overdense to frequencies of 3000 MHz and above.

4 REFLECTION FROM A GRADUAL GRADIENT TO FINITE ELECTRON DENSITY

In this section graphs are presented that give the power reflection coefficient for electromagnetic signals normally incident upon a plasma of finite electron density. The electron density in these cases varies from a value of zero per cubic centimeter very far from the plasma to a value of n_0 deep inside the plasma. The analytical form for the variation of electron density is

$$n_e(z) = \frac{n_0}{1 + e^{-z/\sigma}} \quad (7)$$

The parameter σ characterizes the steepness of the plasma gradient. Vacuum exists at $z = -\infty$; plasma with electron density n_0 exists at $z = +\infty$. For very large negative values of position z ,

$$n_e(z) \approx n_0 e^{+z/\sigma} \quad (8)$$

This analytical form is like that of Sec. 3. Should n_0 be large enough so that the wave does not penetrate to a depth where Eq. (8) is a poor approximation to Eq. (7), then results of this section will reduce to those of Sec. 3 with $\sigma = z_0$ of Sec. 3.

The plasma dielectric constant in the plasma gradient is given by

$$\epsilon(z) = 1 - \frac{X}{1 - iZ} \cdot \frac{1}{1 + e^{-z/\sigma}} \quad (9)$$

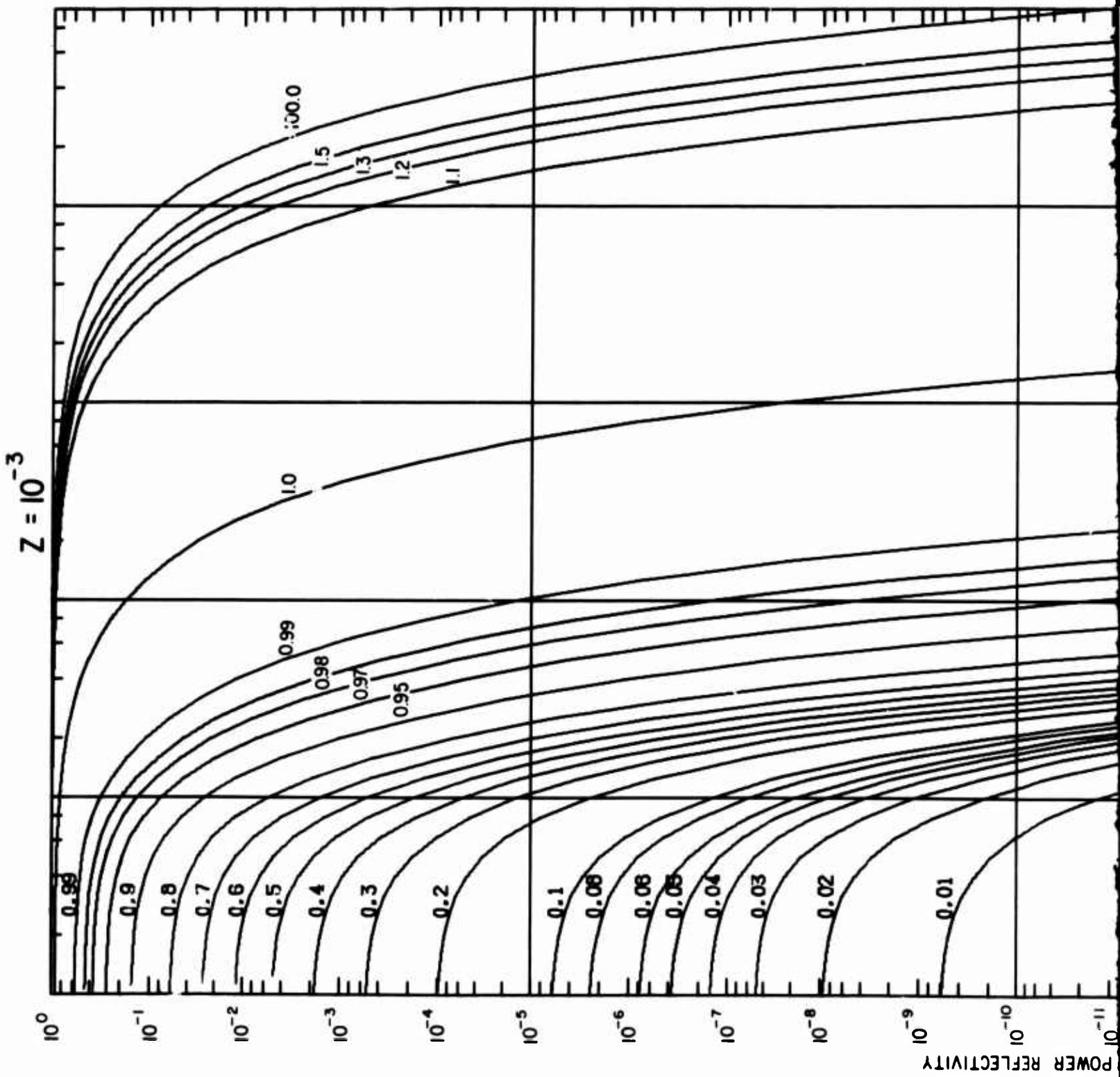
The collision frequency for this form is constant as a function of position in the plasma gradient. This condition approximates plasmas in the lower atmosphere. The power-reflection coefficient for this analytical gradient form, given in Ref. 3, is

$$R = \left| \frac{1 - \sqrt{\epsilon_2}}{1 + \sqrt{\epsilon_2}} \right|^2 \left| \frac{\Gamma[1 + ik\sigma(\sqrt{\epsilon_2} + 1)]}{\Gamma[1 + ik\sigma(\sqrt{\epsilon_2} - 1)]} \right|^4 \quad (10)$$

where ϵ_2 is the dielectric constant from Eq. (9) at z equal to plus infinity. The parameter k is equal to $2\pi/\lambda$. The first factor on the right-hand side of Eq. (10) is the familiar Fresnel equation for power reflection from an abrupt interface between vacuum and plasma. For this reason we refer to this factor as the Fresnel term in this report. The second factor, involving the gamma function ratio, introduces the effect of the change in electron density occurring over a finite distance. It is clear that if σ goes to zero, meaning an infinitely sharp transition, then Eq. (10) is left with only the Fresnel term, as is necessary. The Appendix indicates in detail how Eq. (10) was evaluated and the accuracy that we expect in our plots.

The power-reflection coefficient is plotted as a function of Z , f_p/f , and σ/λ in Figs. 9 and 10. In Fig. 9(a) through 9(m) the logarithm of the power-reflection coefficient is plotted versus the logarithm of the ratio σ/λ . Each graph page is for a fixed value of Z for values of Z from 10^{-3} to 10^{+3} . The power-reflection coefficient was computed and plotted versus σ/λ for a fixed f_p/f ratio, where f_p is the plasma frequency and f is the radar frequency. The choices of values for this ratio that were plotted were made for each graph so as to minimize gaps between adjacent curves. Thus, different choices for the values of f_p/f will appear on various graphs. The power-reflection coefficient was computed for Z and a fixed value of X , which is $(f_p/f)^2$. The computation was performed for values of σ/λ that were stepped in the ratio of $(10)^{1/10}$; in this way, ten points were plotted at even intervals for each power-of-10 change along the logarithmic abscissa.

The general behavior of all of these curves of power reflectivity versus σ/λ , as presented in Fig. 9, is similar. The power reflectivity is constant at the Fresnel value as a function of σ/λ for small values of σ/λ . Then the reflectivity proceeds to decrease extremely rapidly as the σ/λ ratio continues to increase. For fairly low values of Z and large values of σ/λ , we note that the power reflectivity changes from almost no reflectivity for $f_p/f \approx 0.9$, to a rather substantial reflectivity for $f_p/f \approx 1.1$. This extreme change in reflectivity with f_p/f



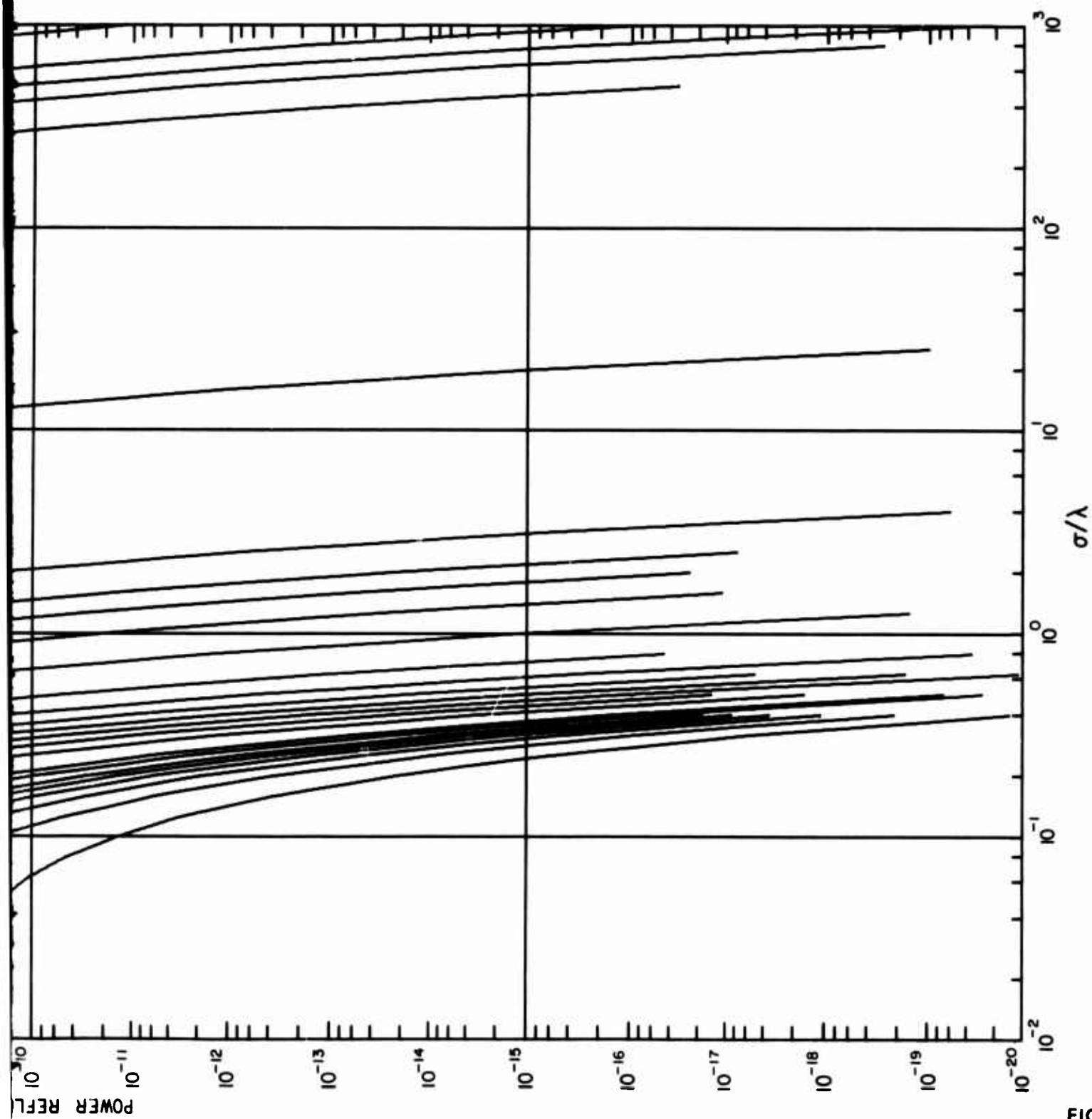
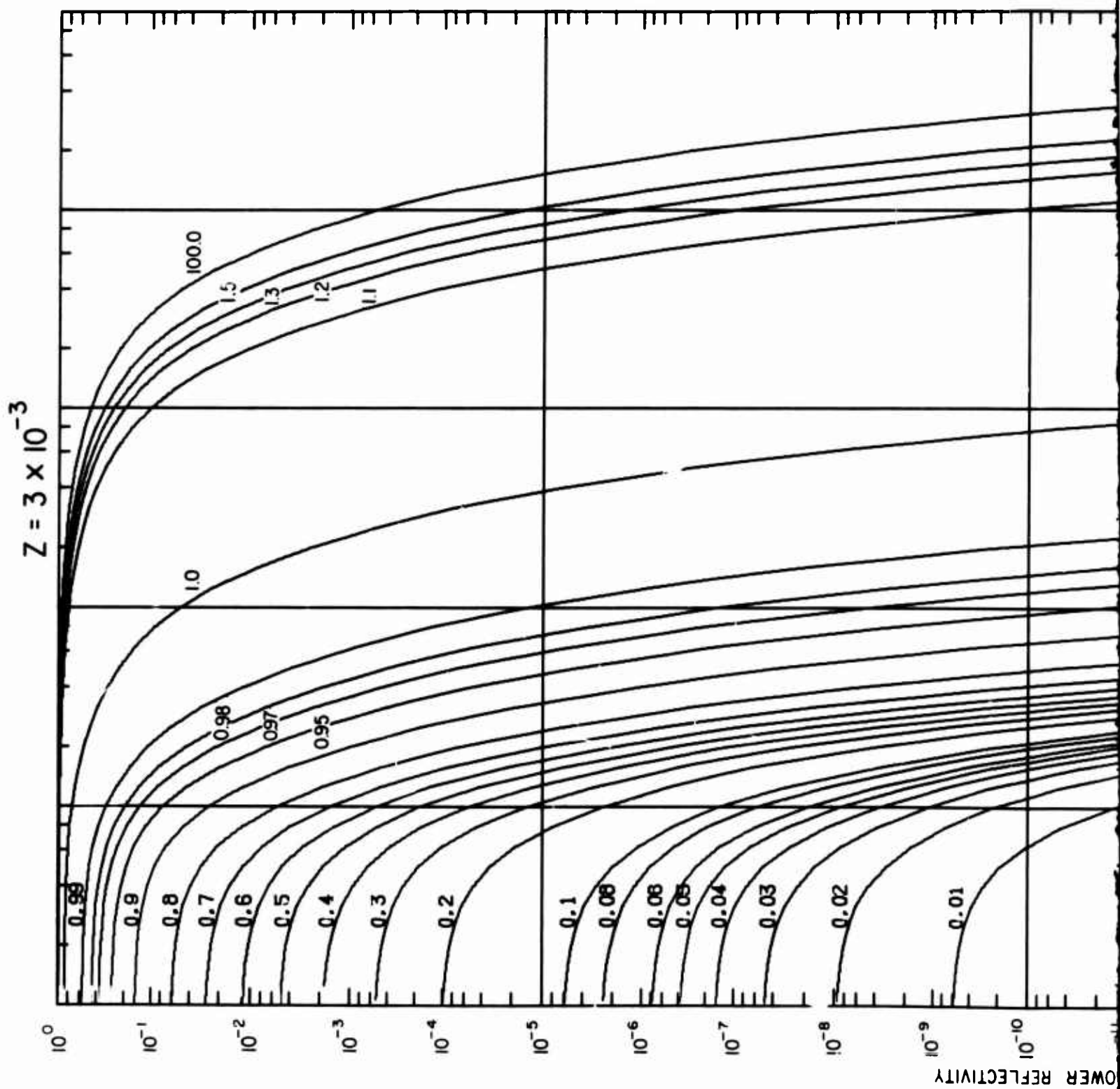


FIG. 9(a)

FIG. 9(a) POWER-REFLECTION COEFFICIENT vs. THE RATIO σ/λ FOR VALUES OF Z INDICATED AT TOP OF FIGURE

B



D

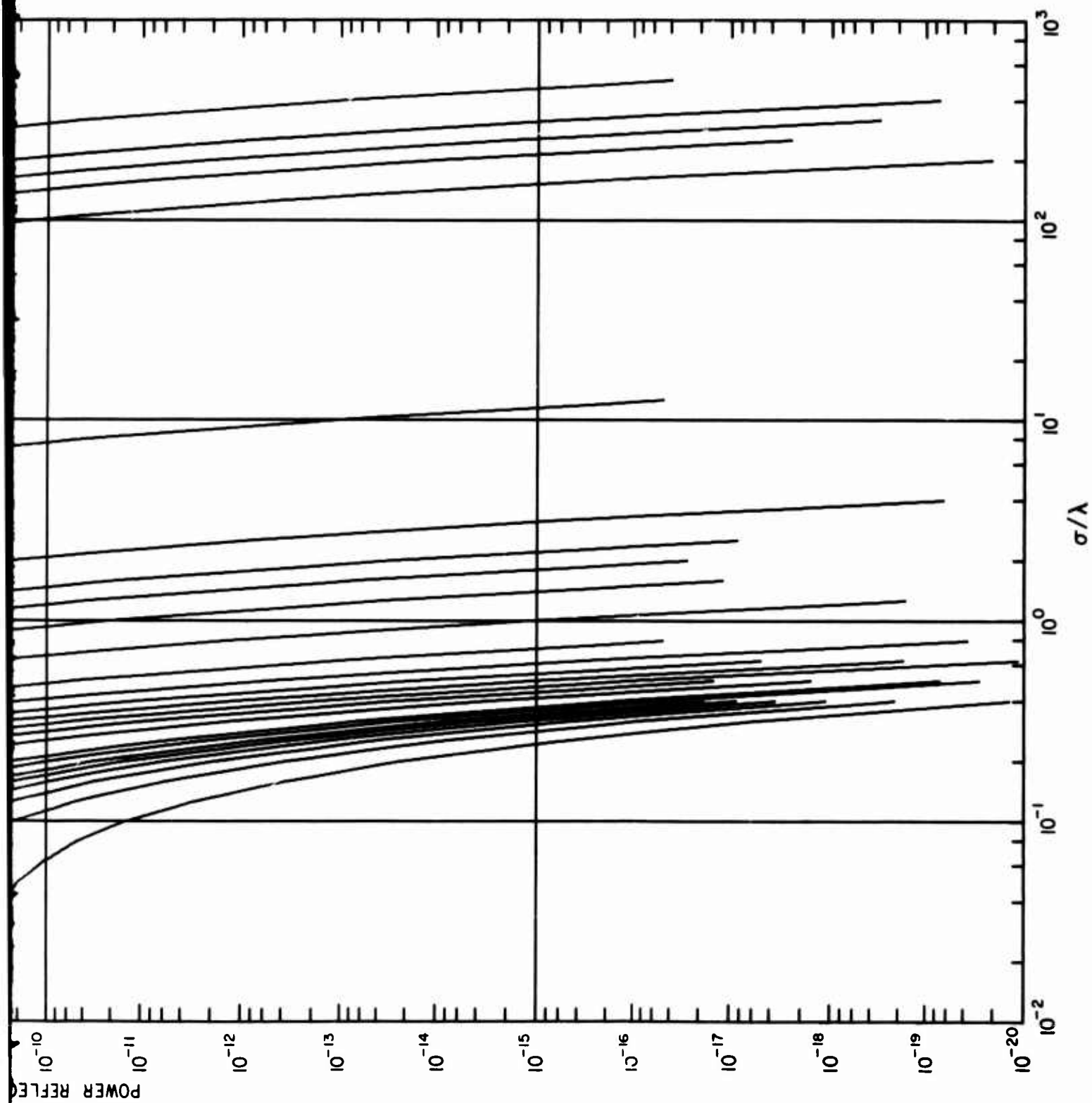
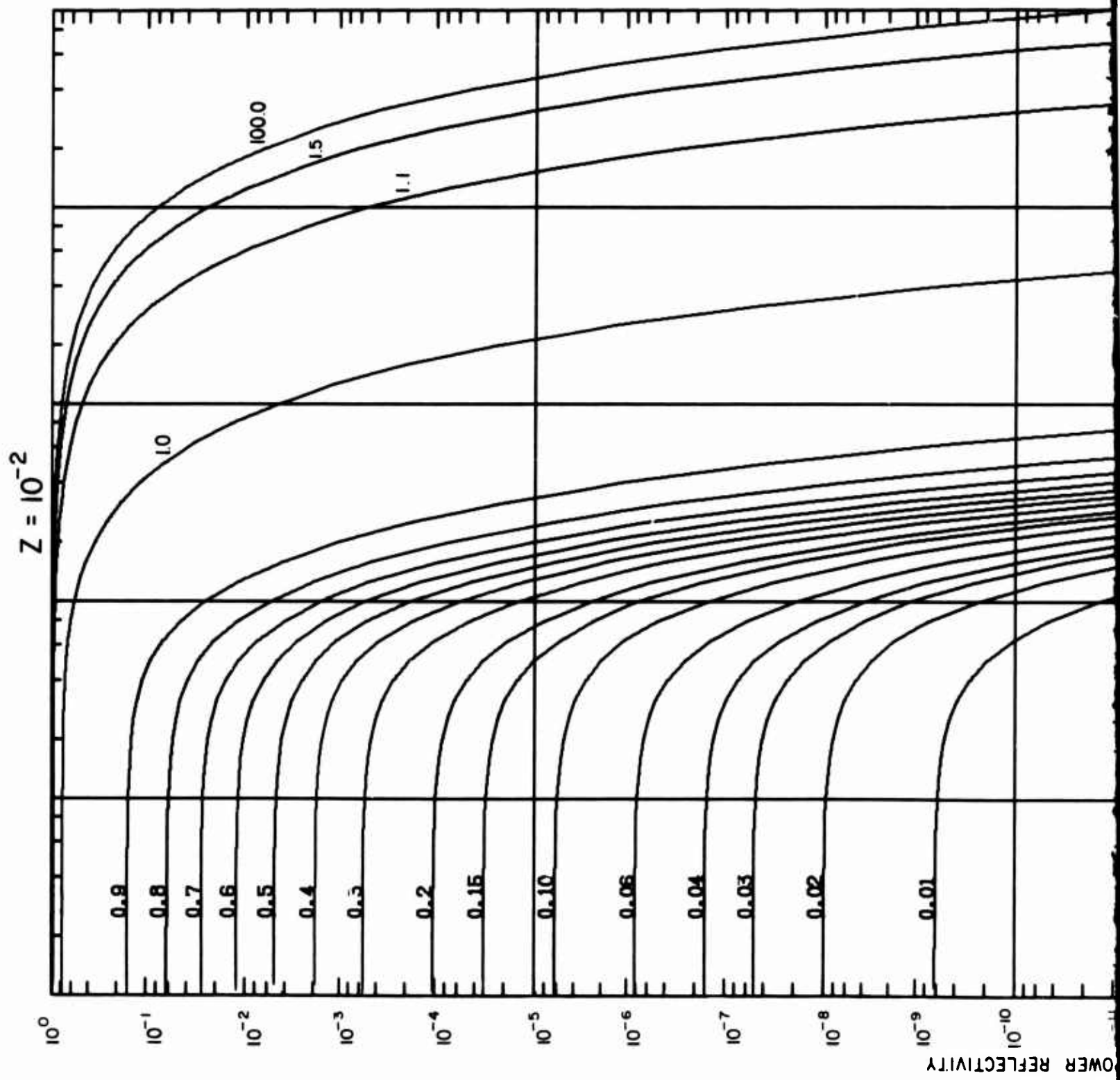


FIG. 9(b)

B



T

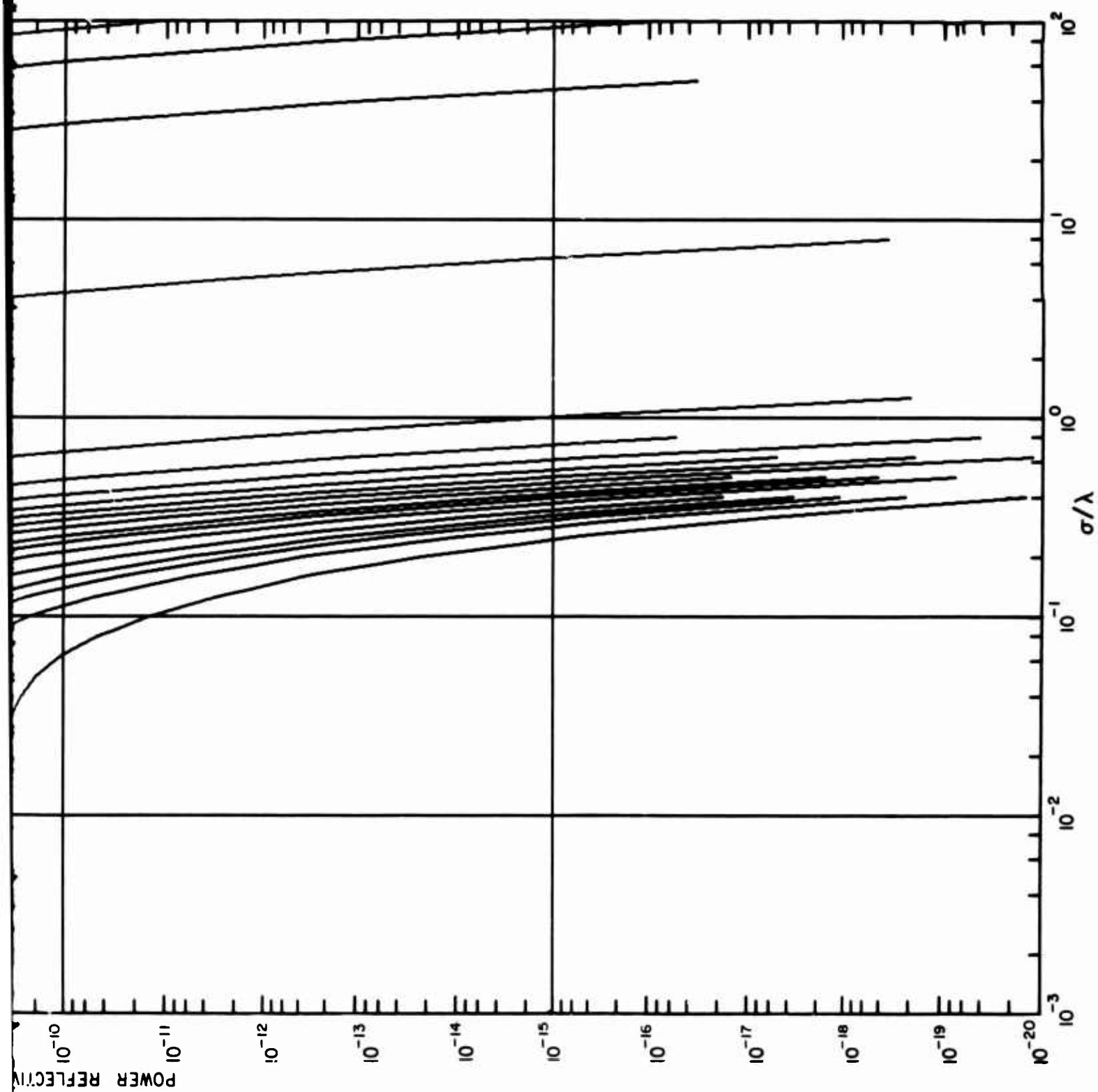
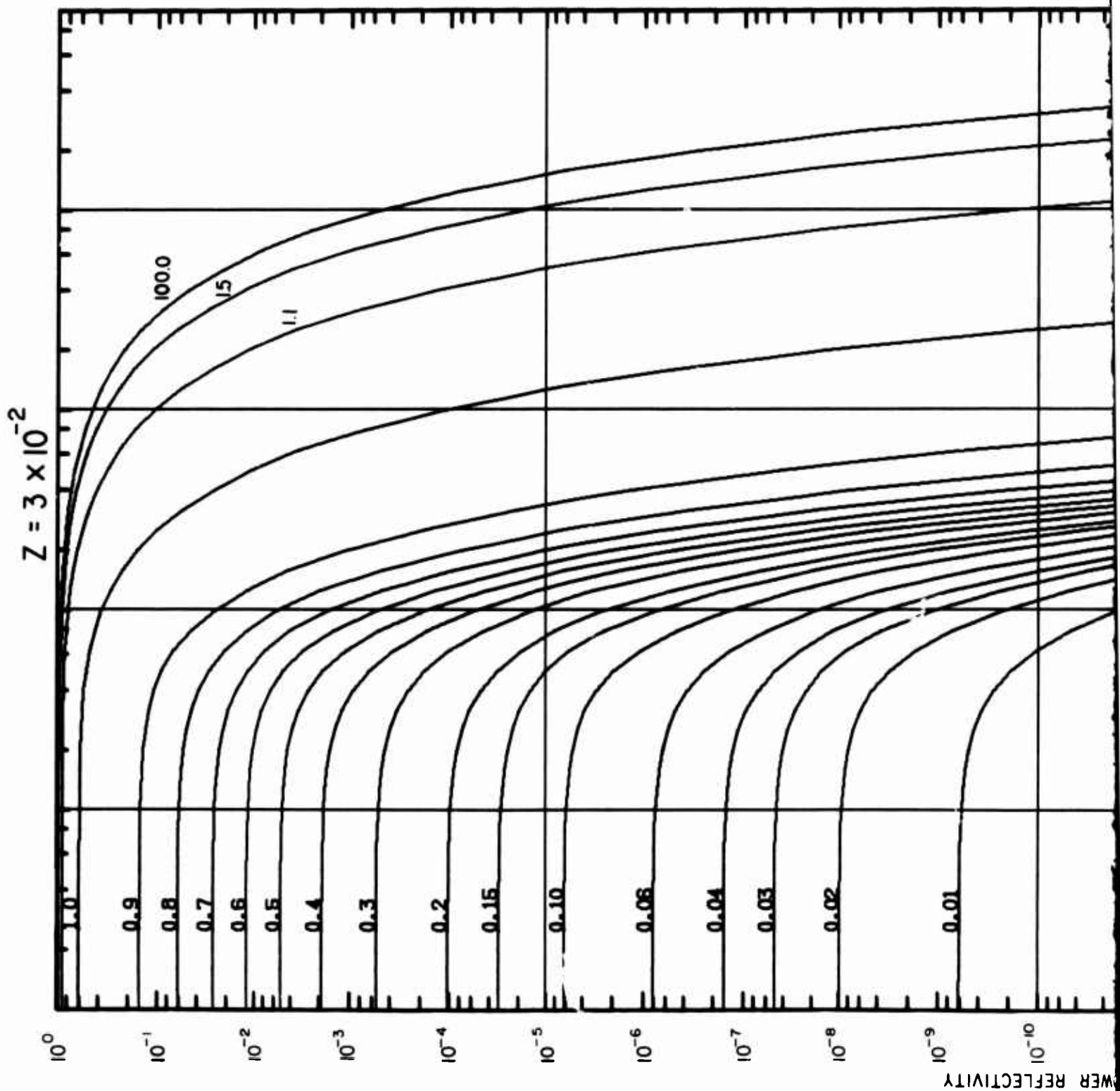


FIG. 9(c)

B



A

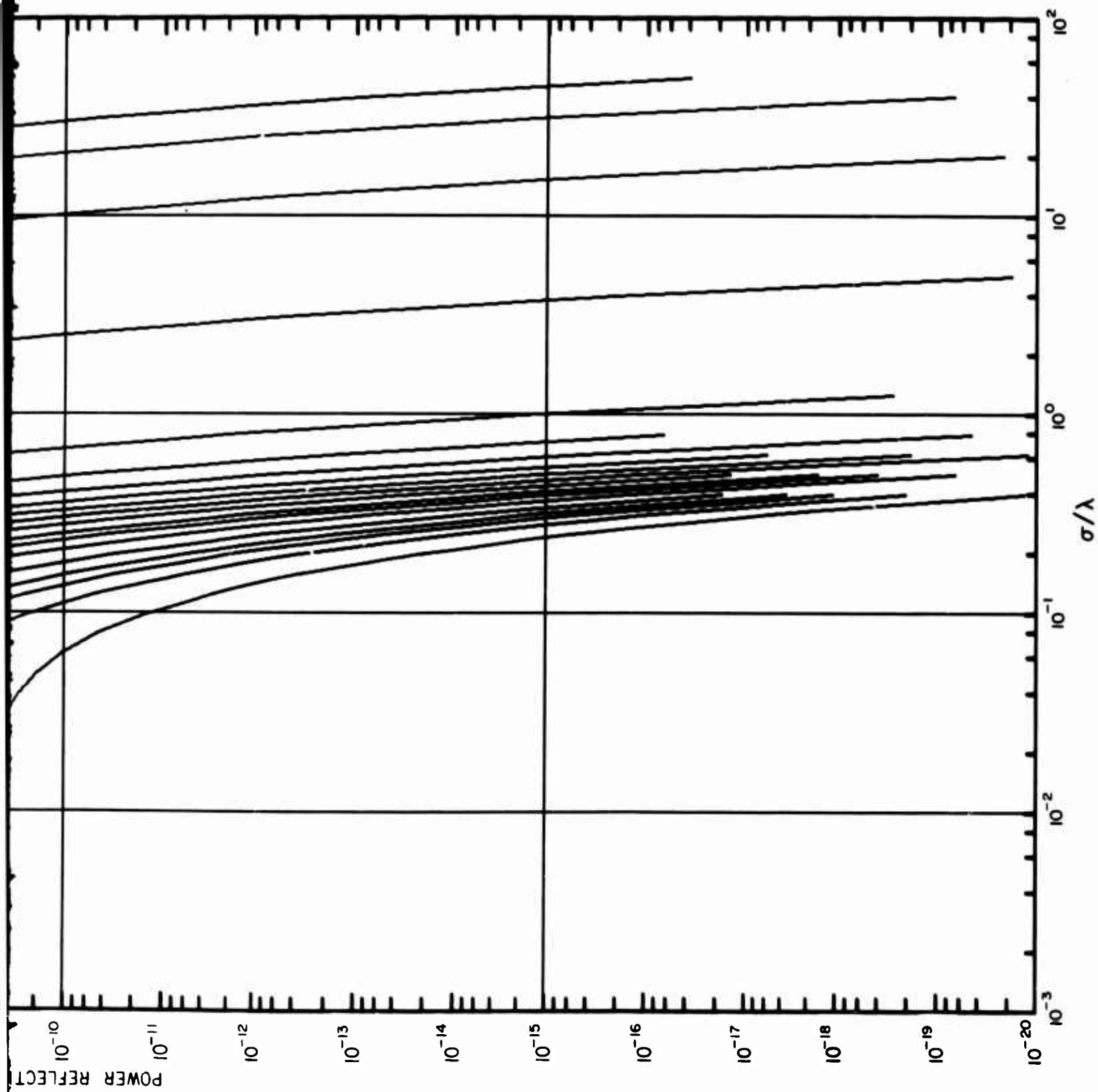
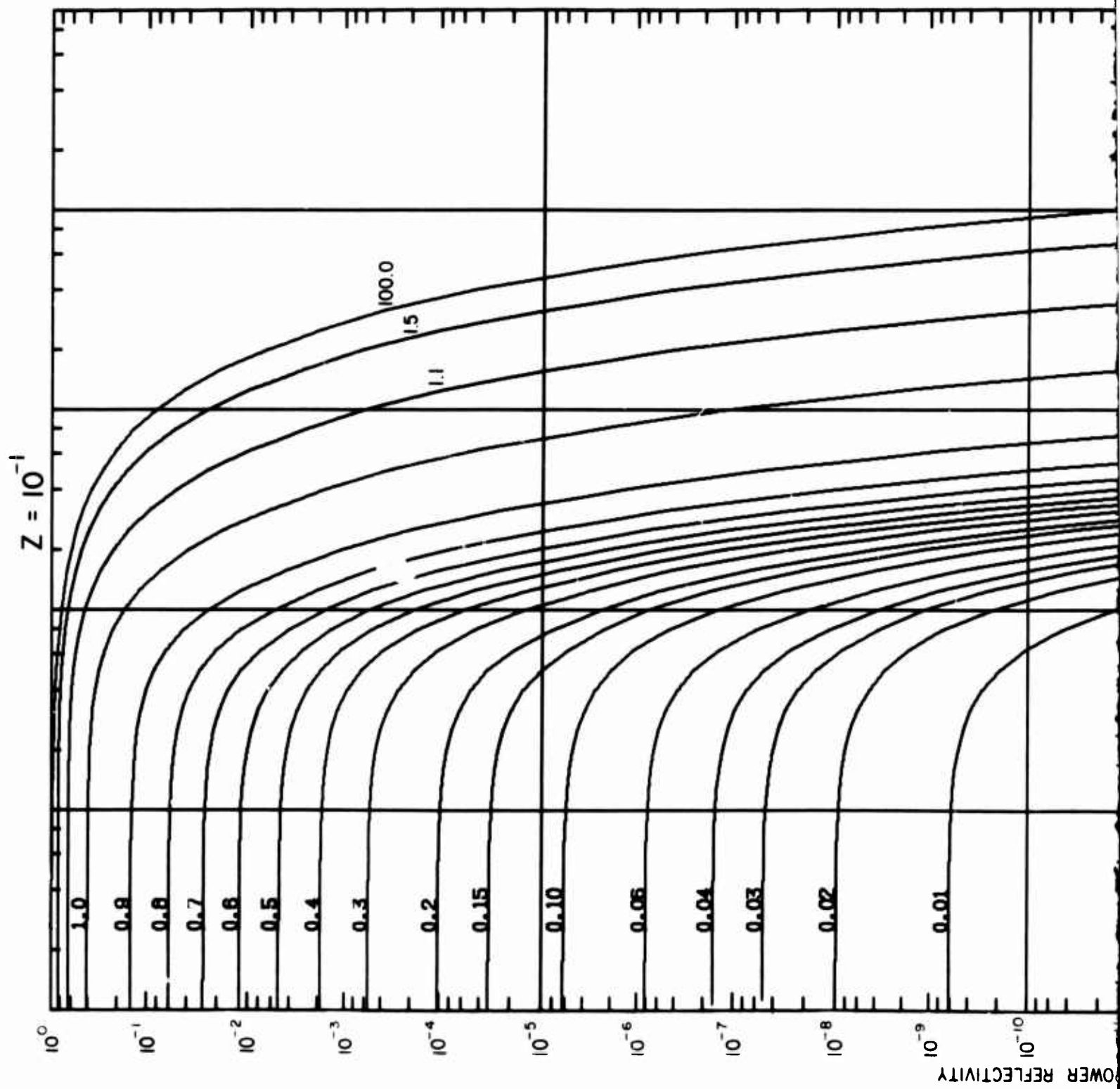


FIG. 9(d)



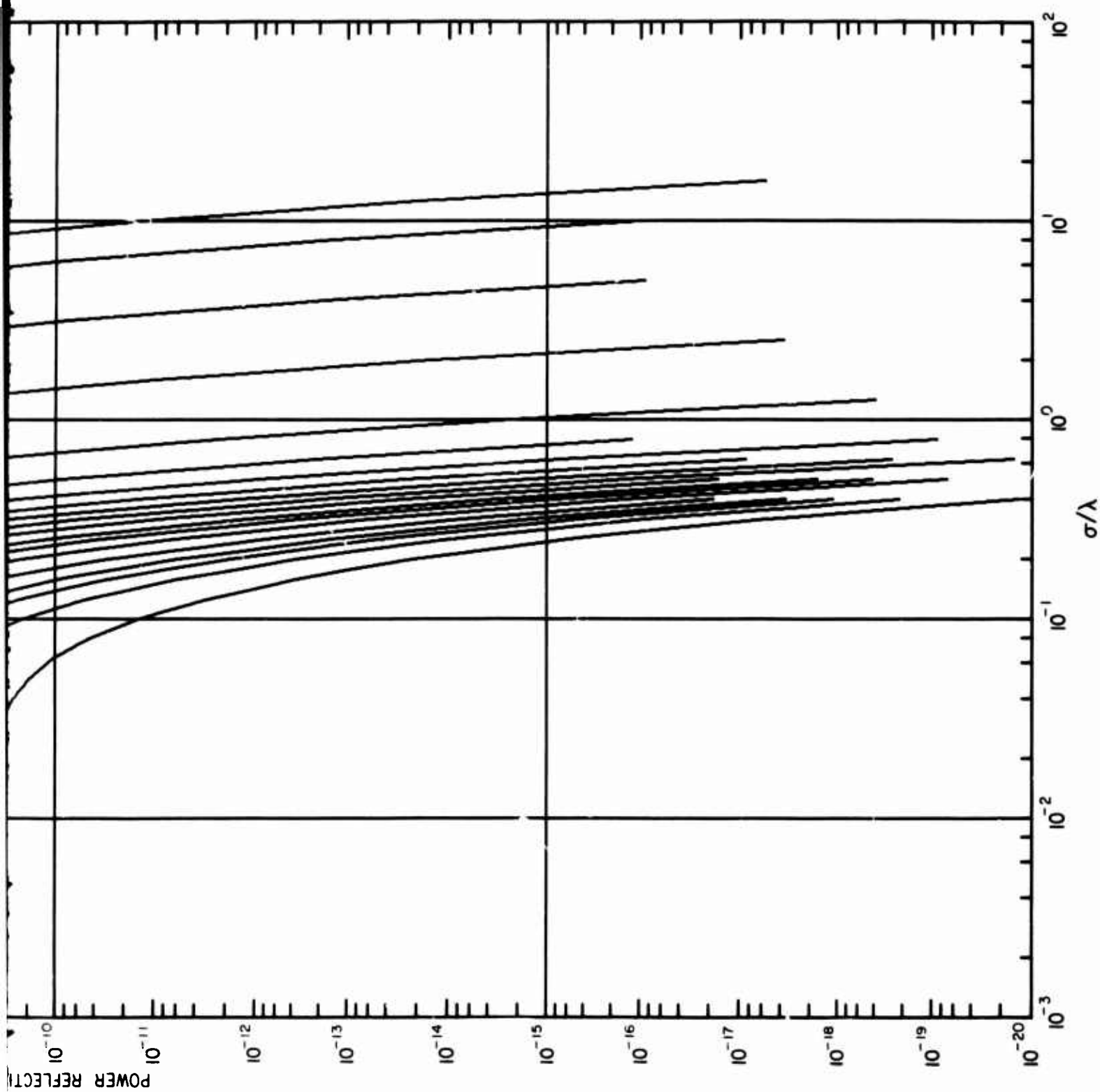
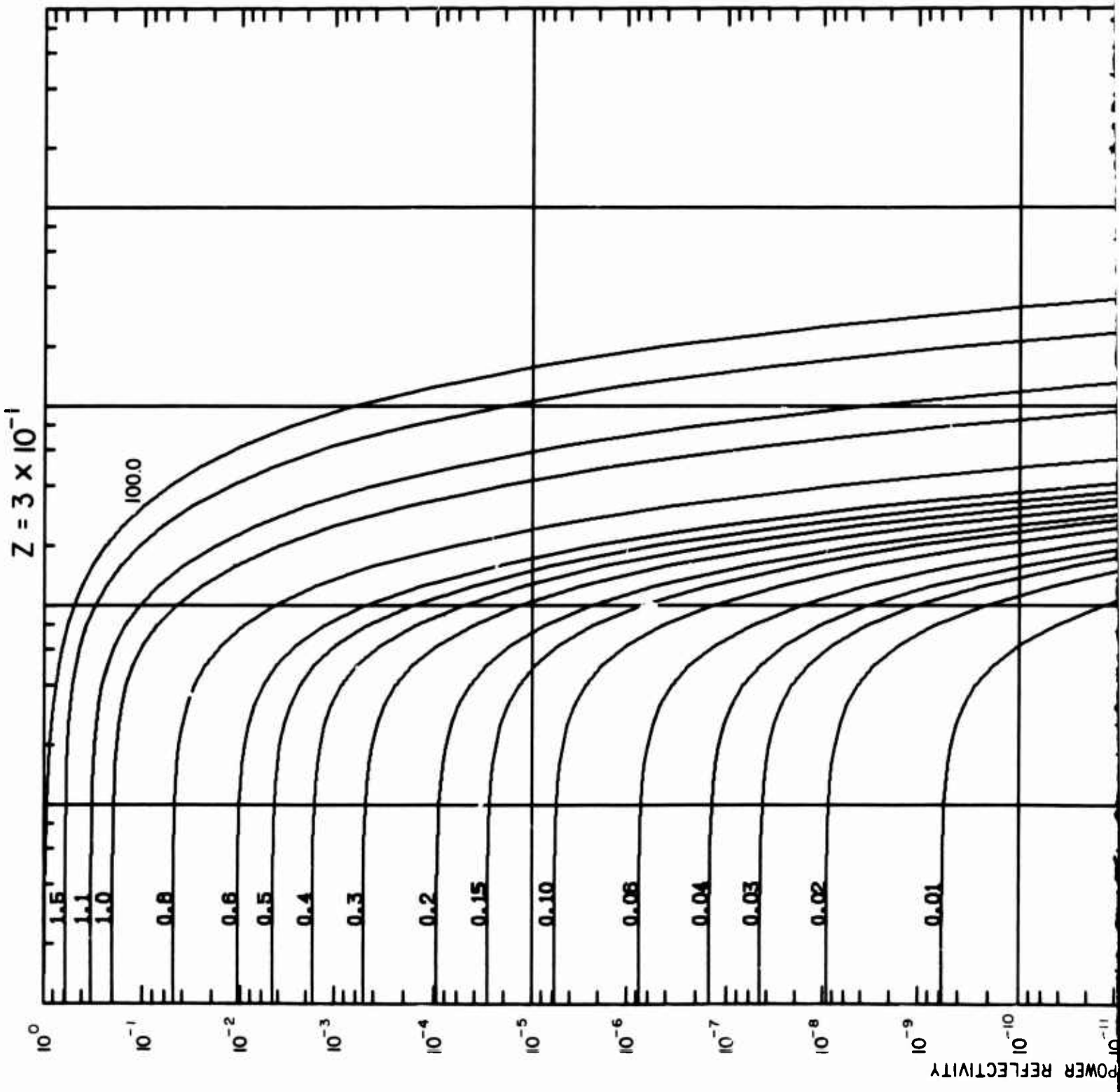


FIG. 9(e)



A

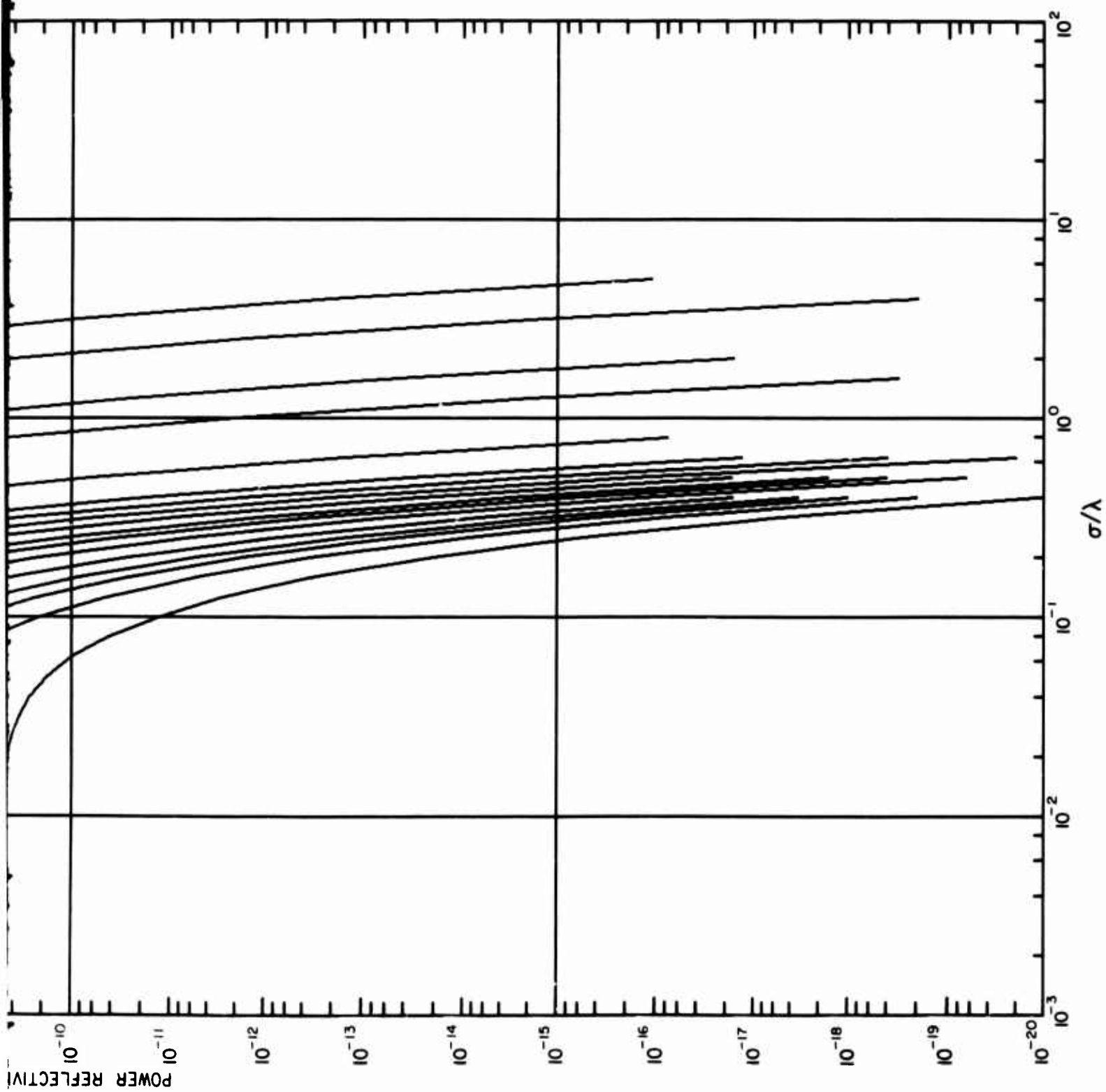
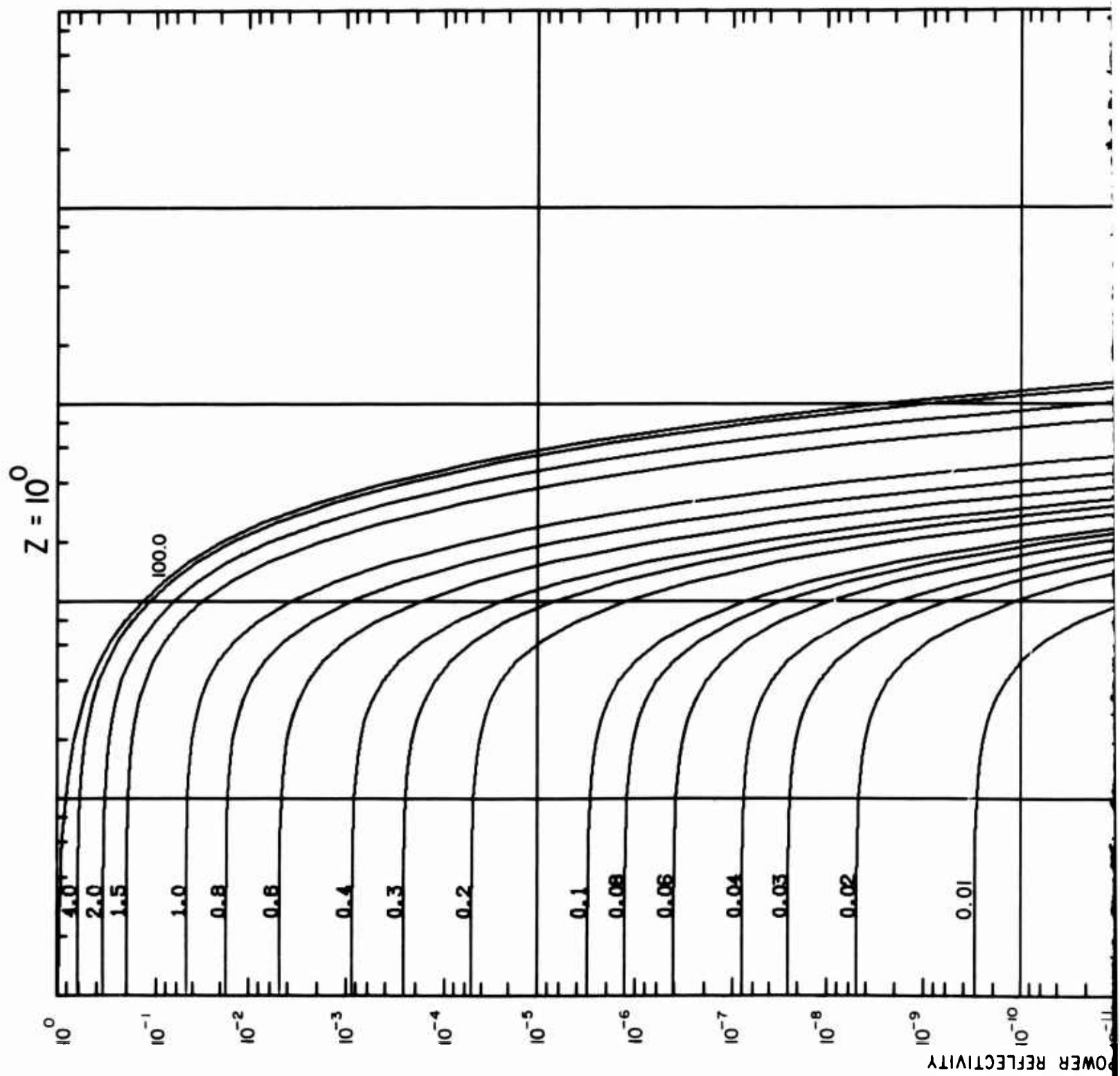
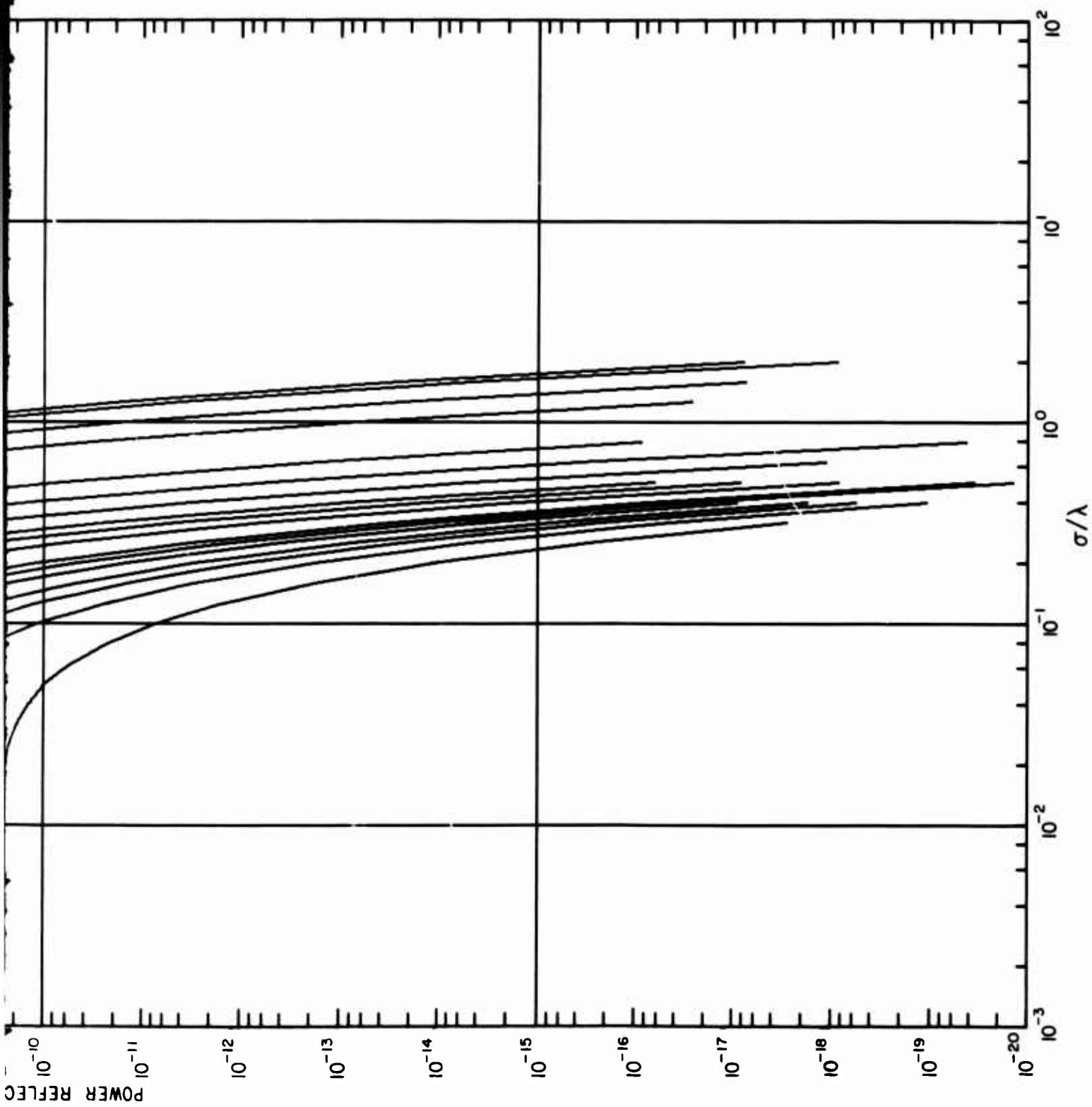
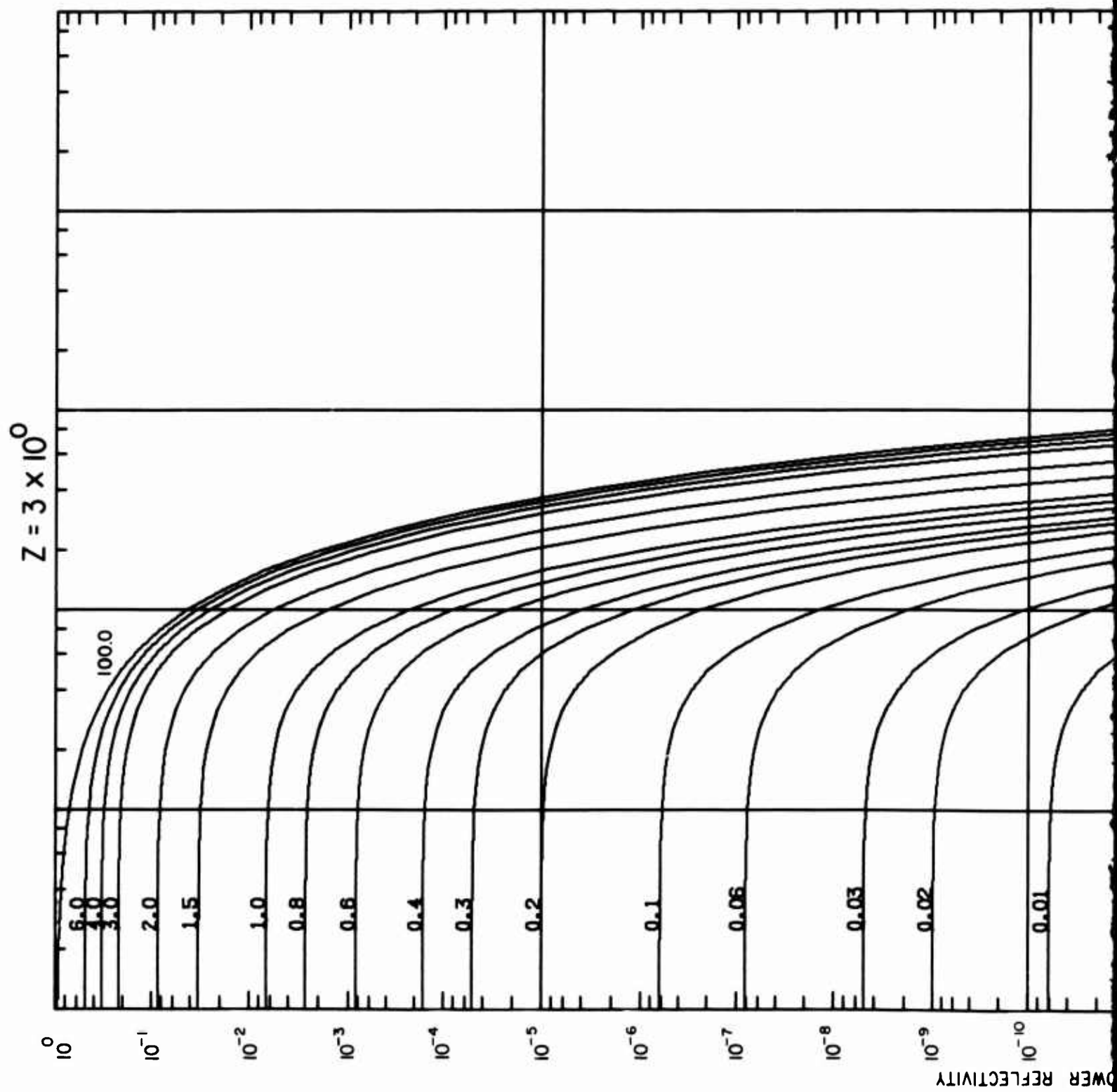


FIG. 9(f)



A





7

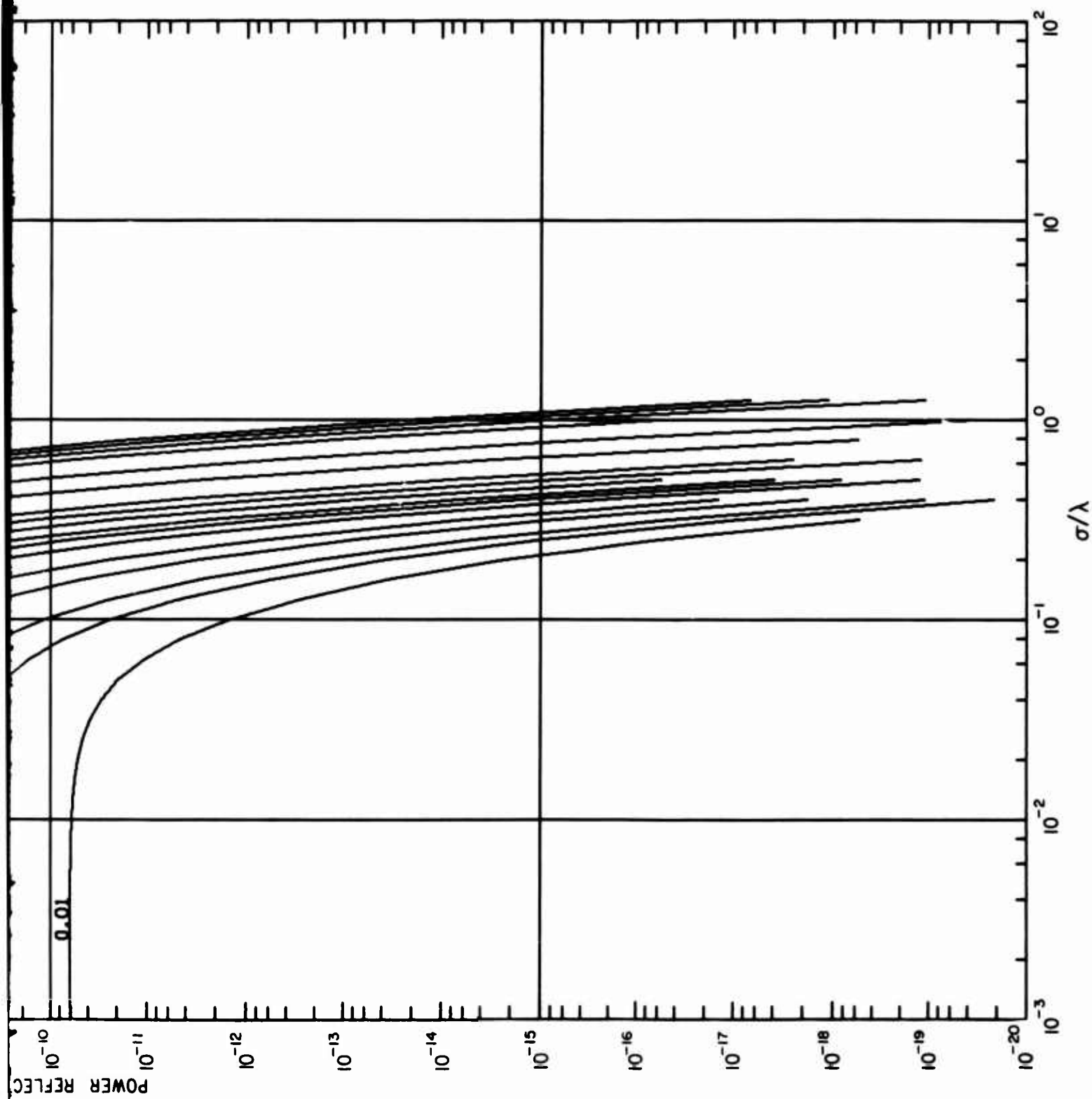
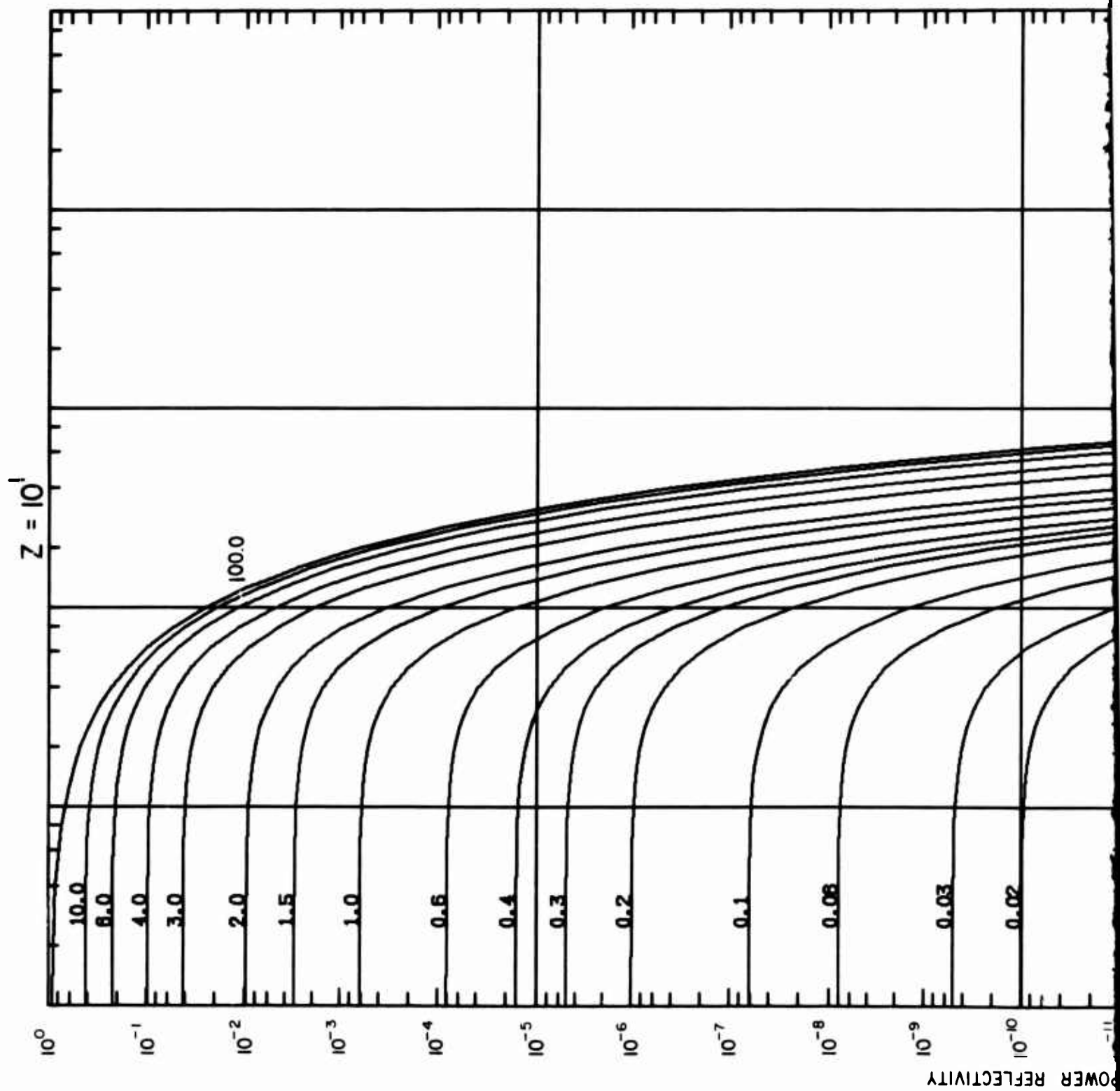


FIG. 9(h)

B



A

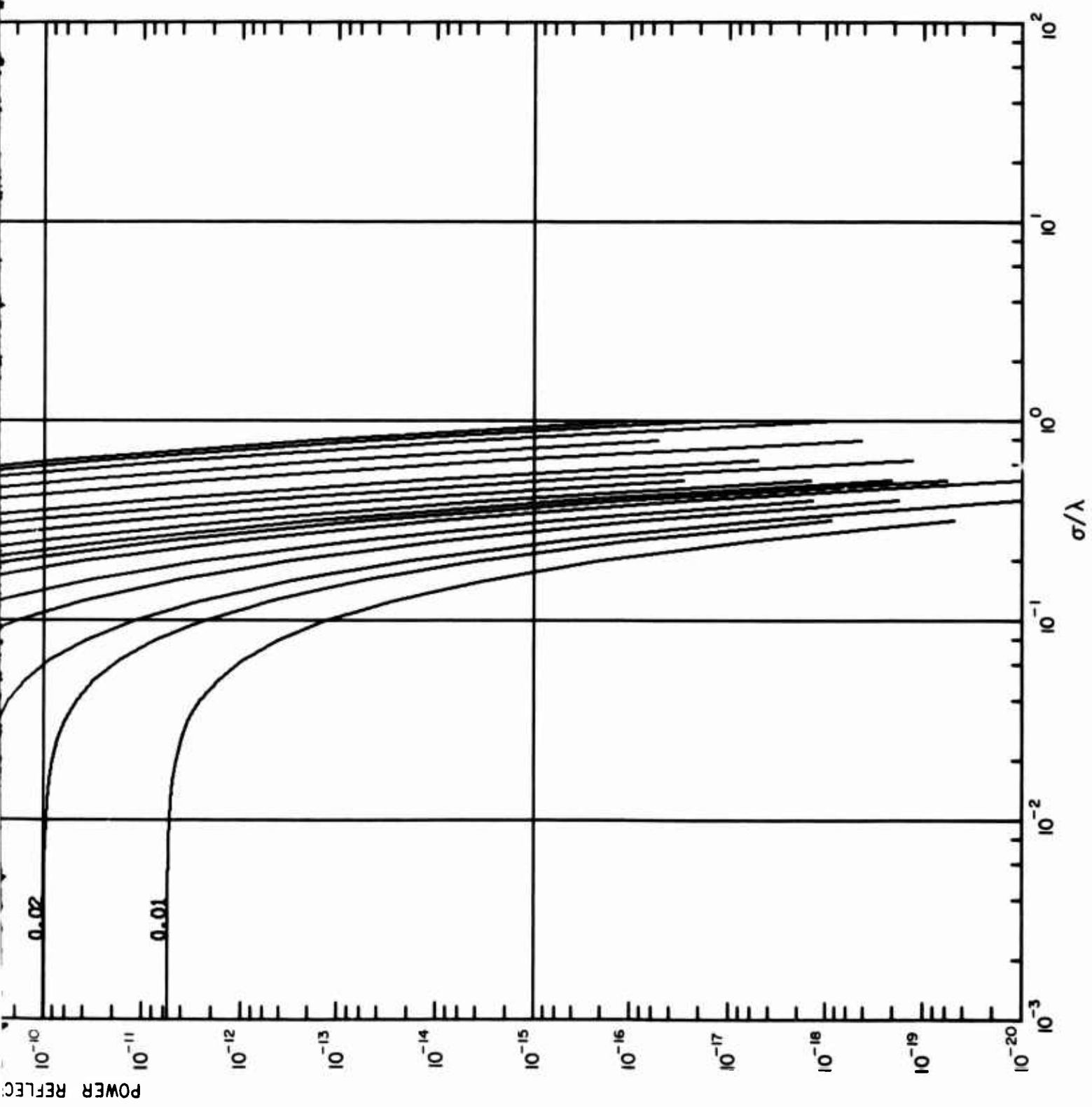
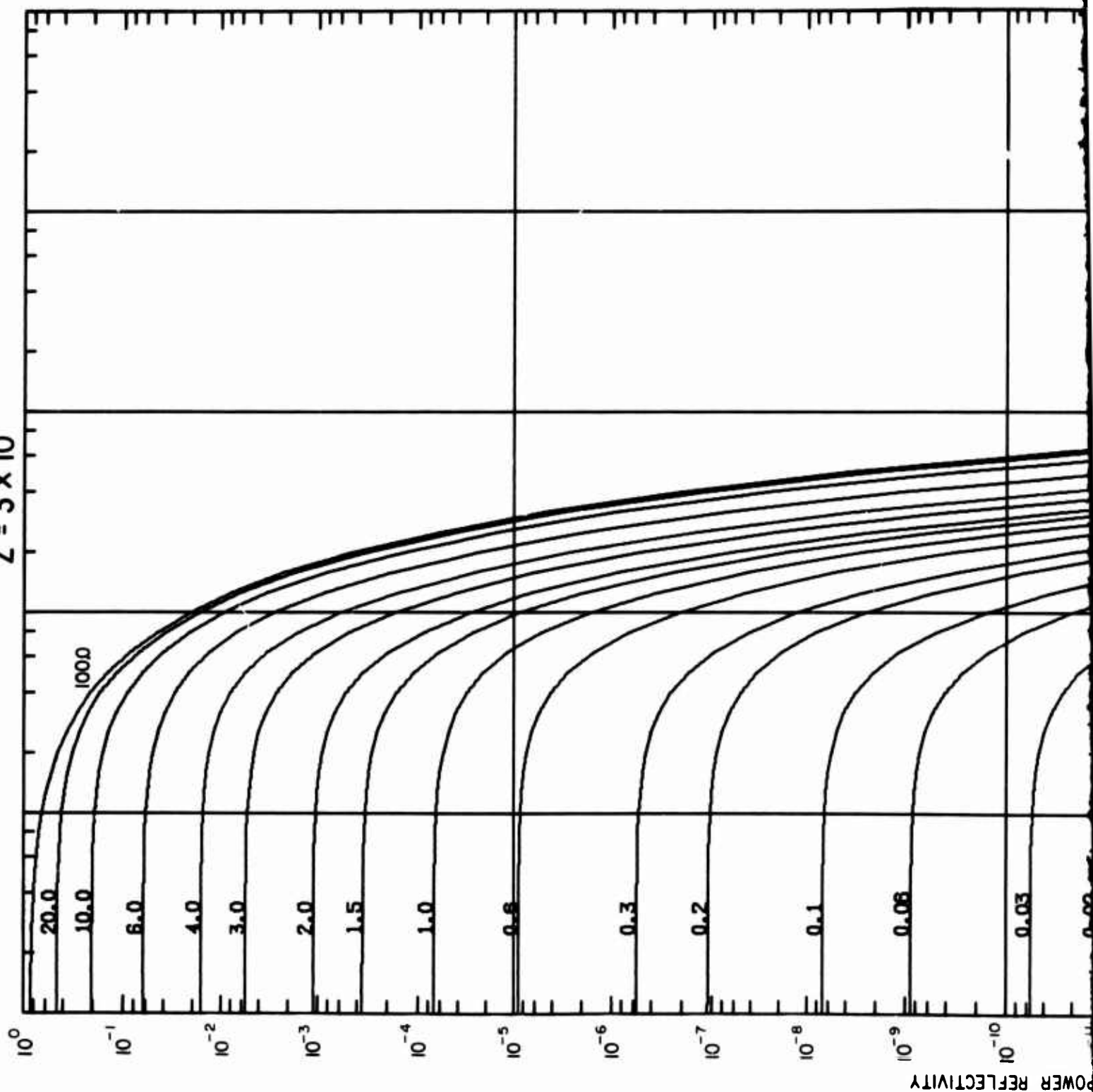


FIG. 9(i)

B

$Z = 3 \times 10^1$



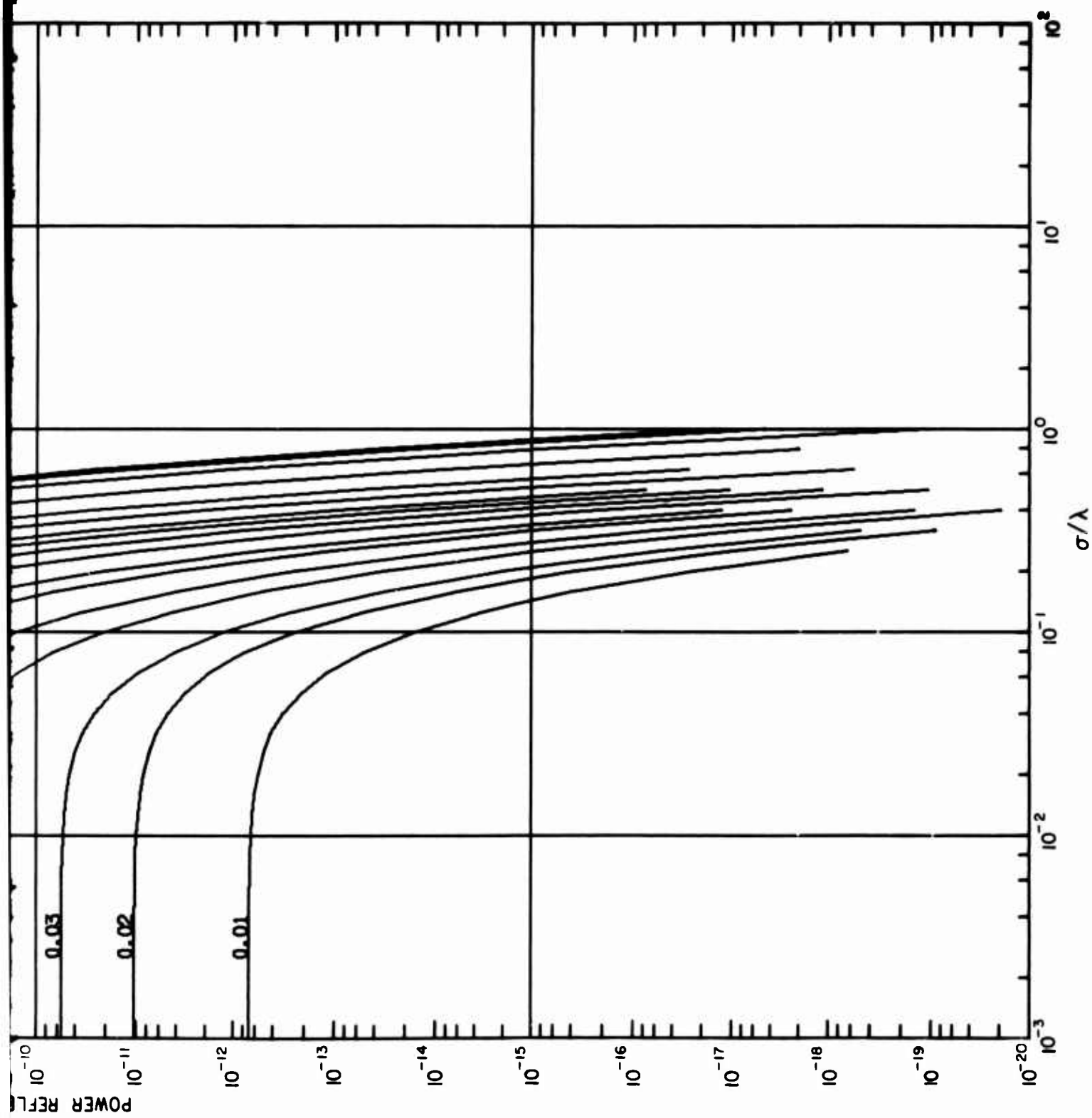
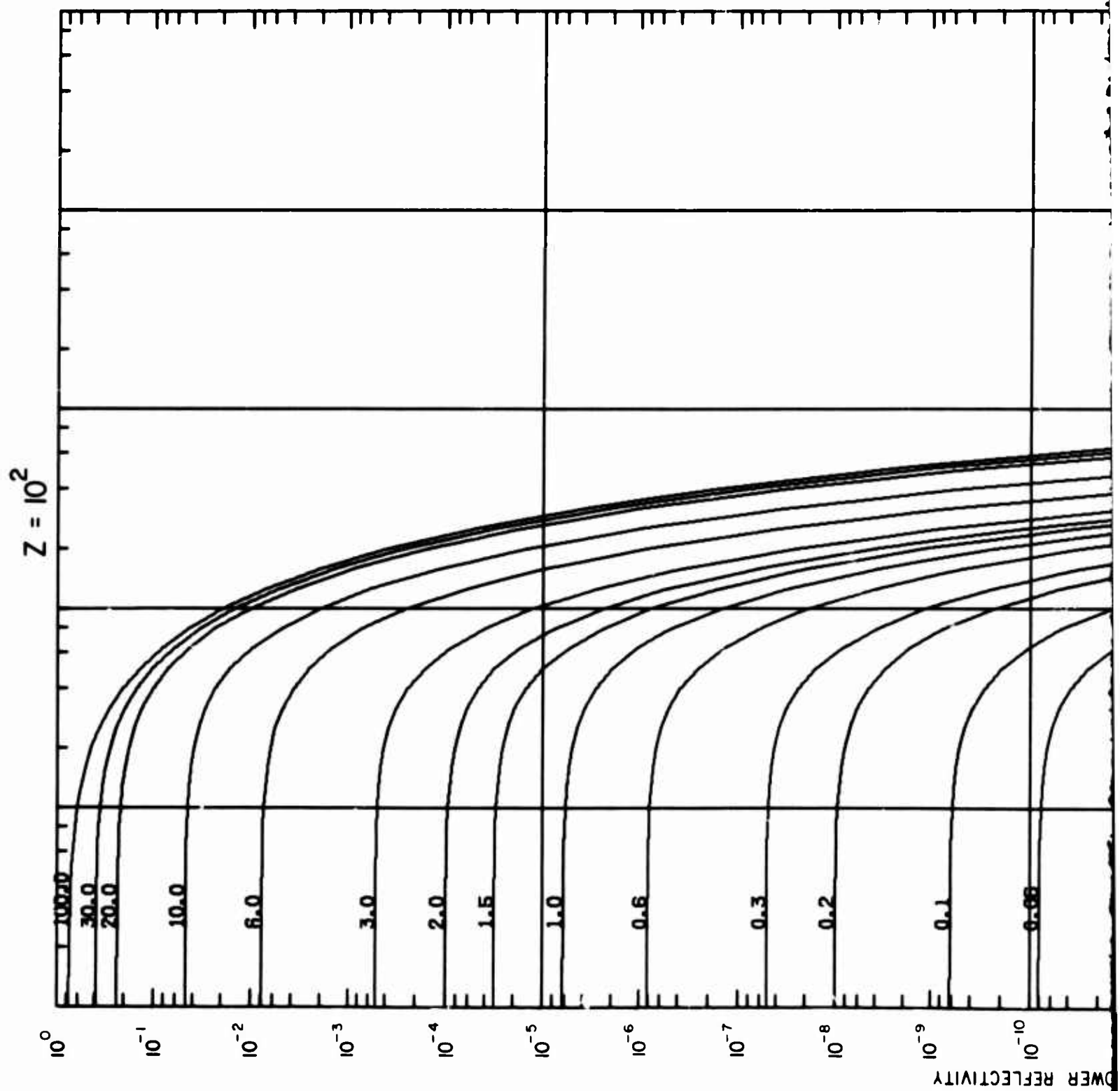


FIG. 9(j)



I

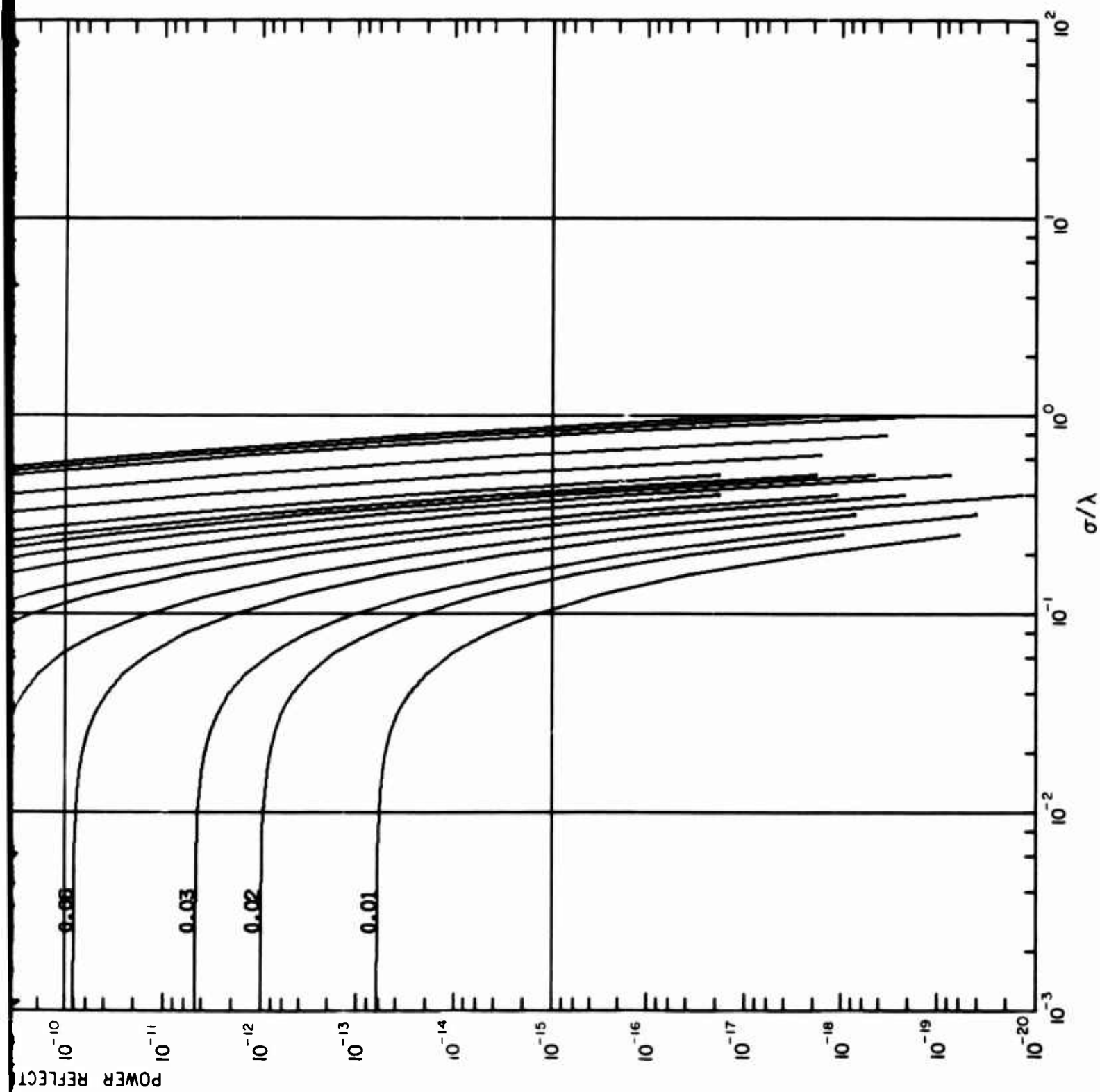
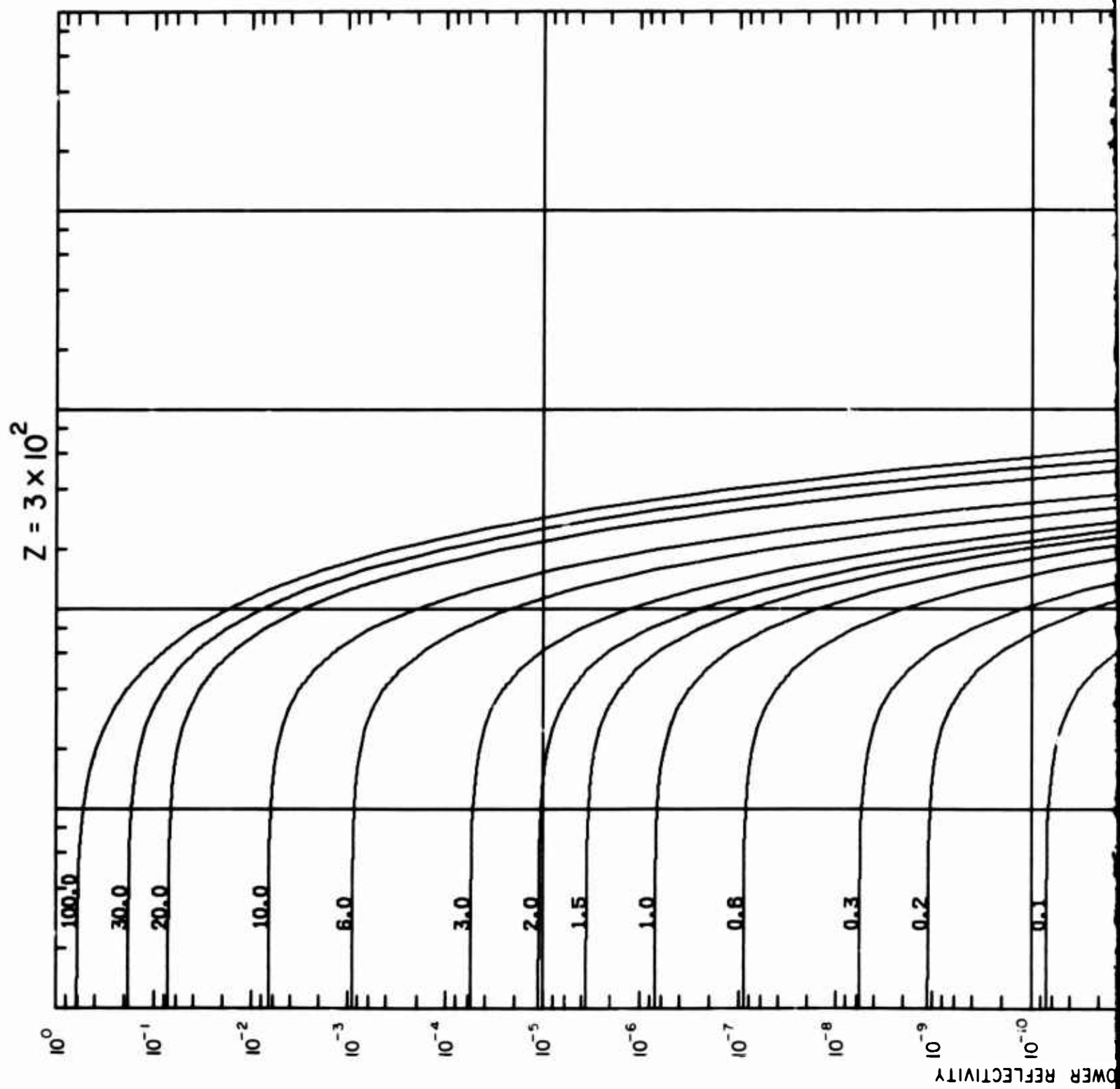


FIG. 9(k)

B



H

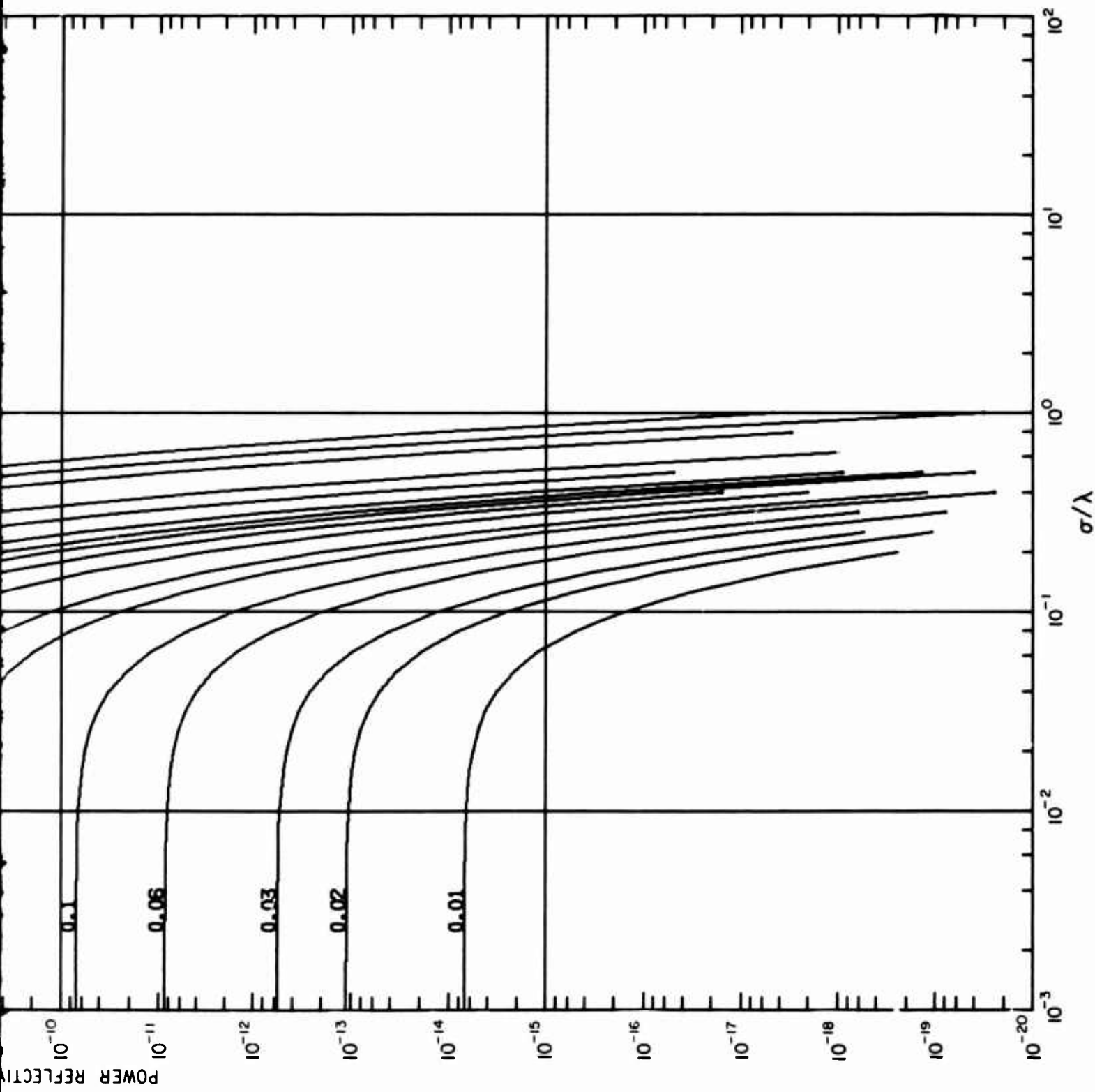
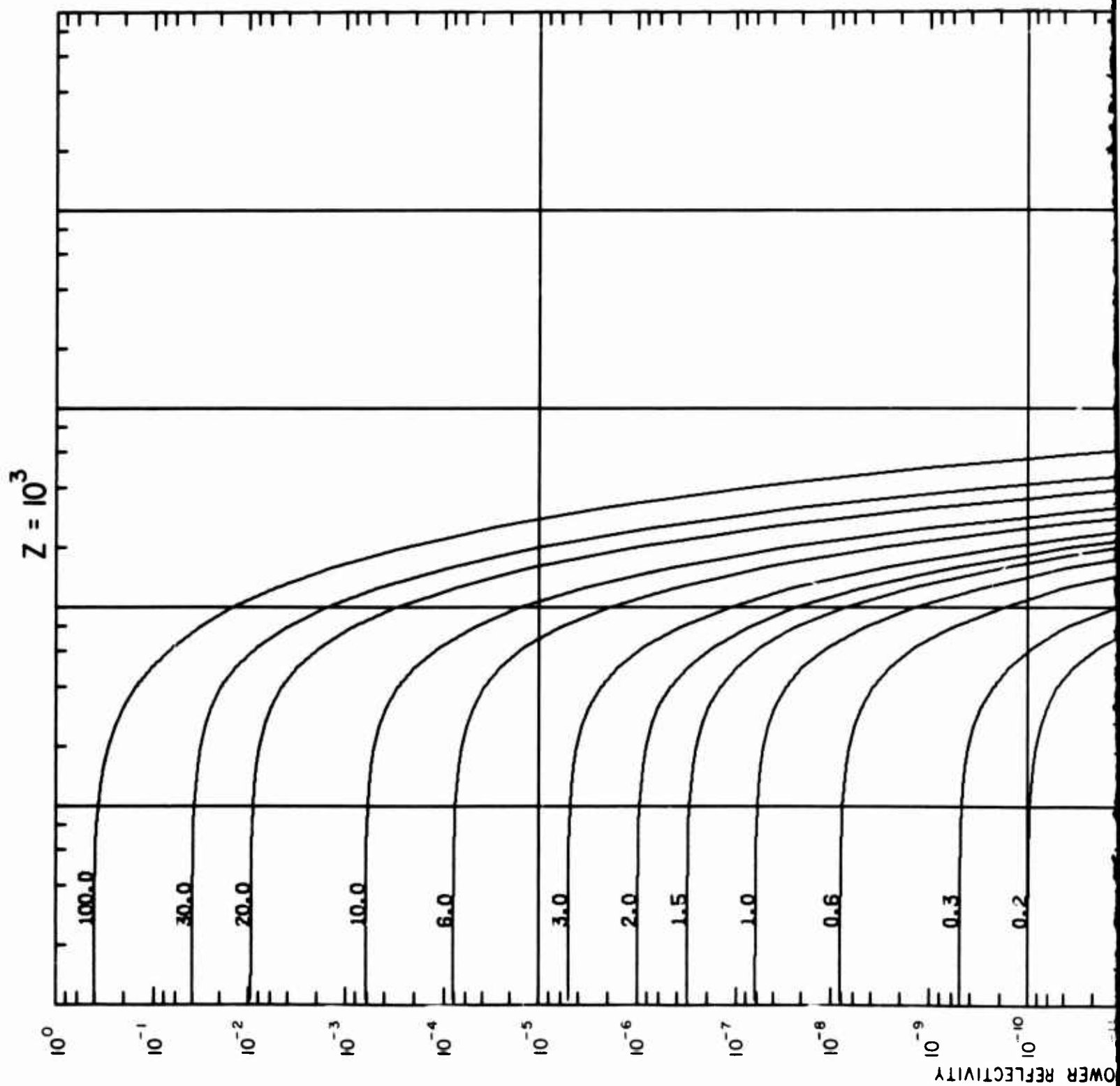


FIG. 9(I)

B



H

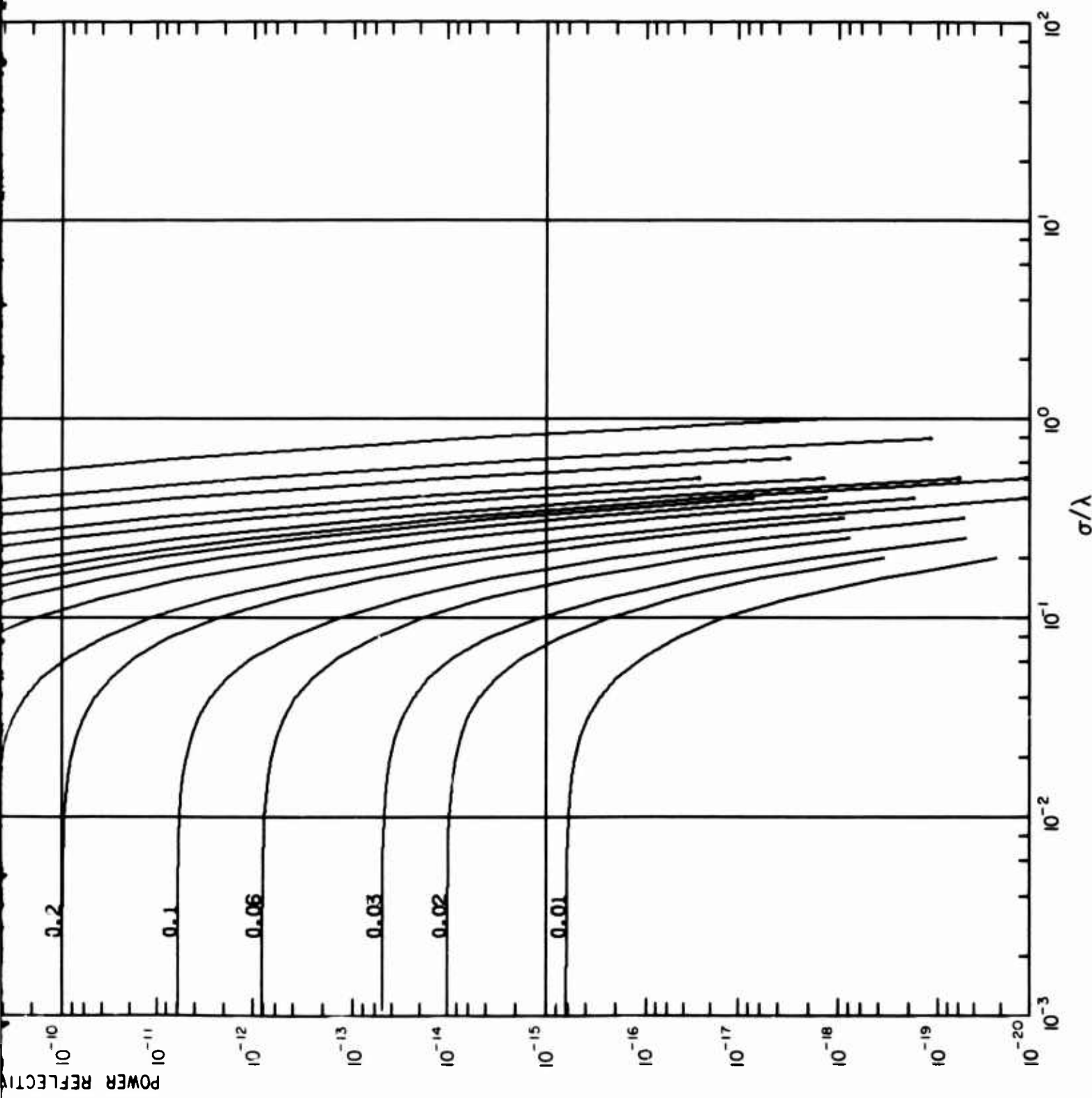
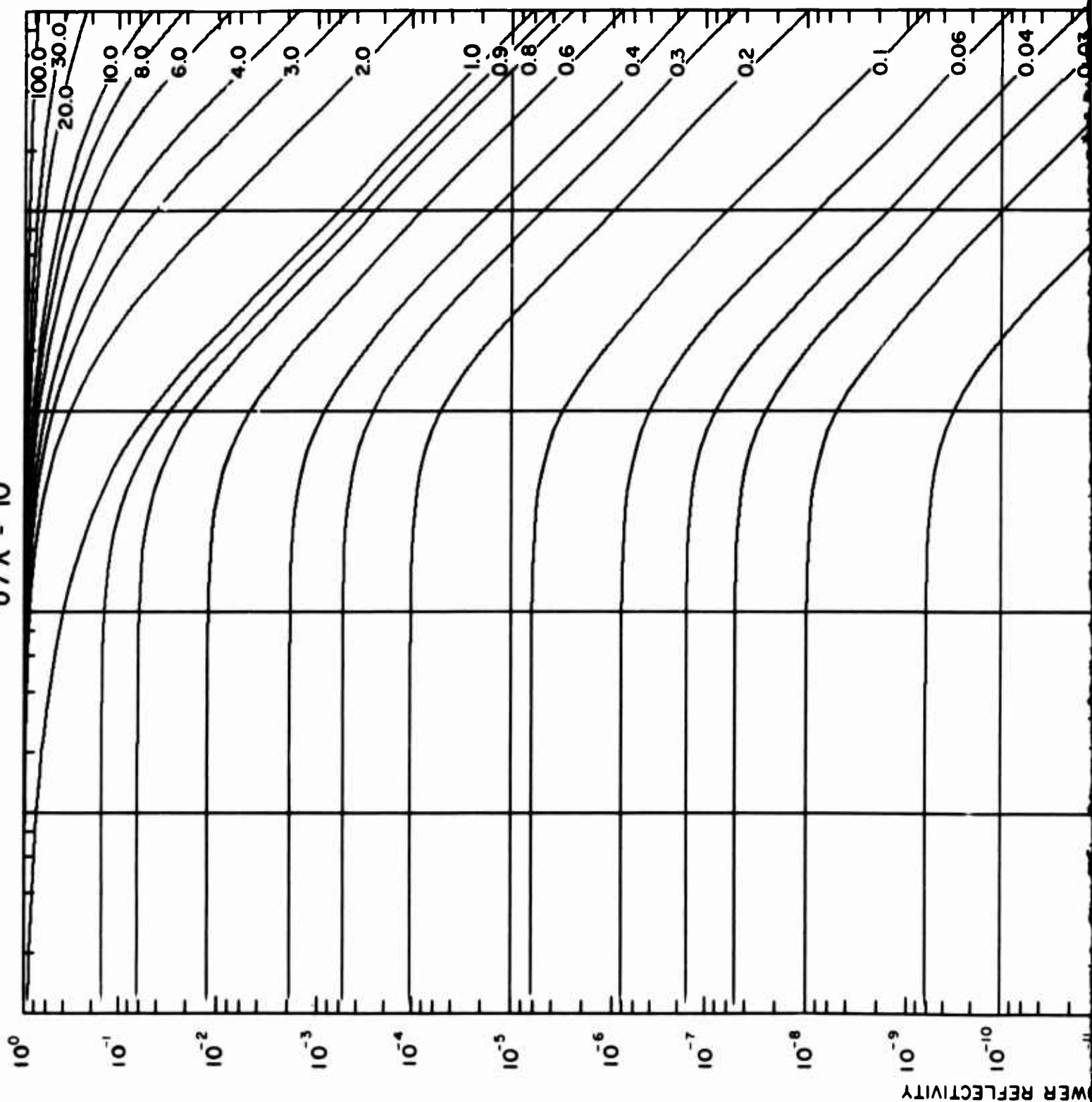


FIG. 9(m)

B

$$\sigma/\lambda = 10^{-3}$$



A

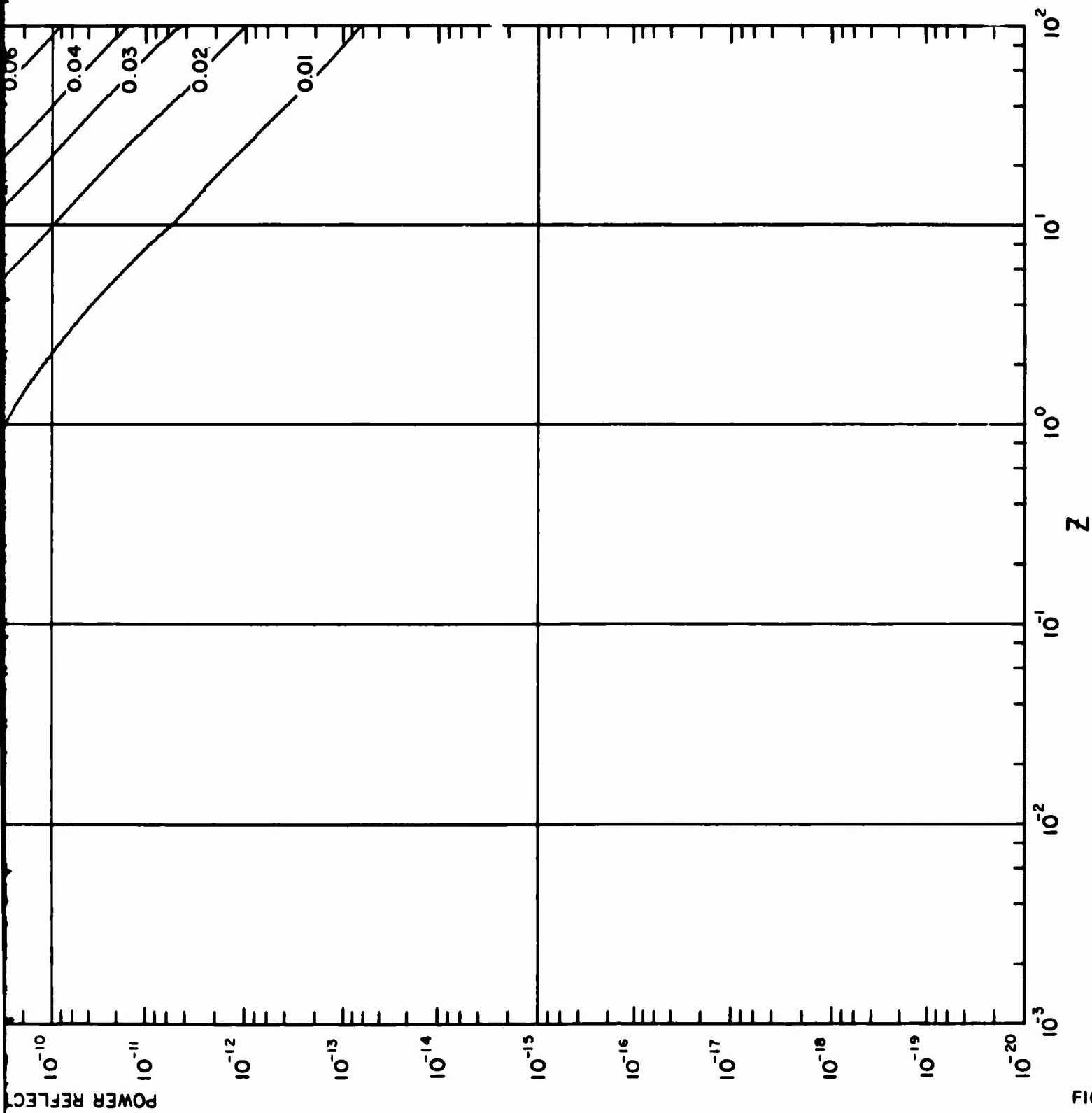
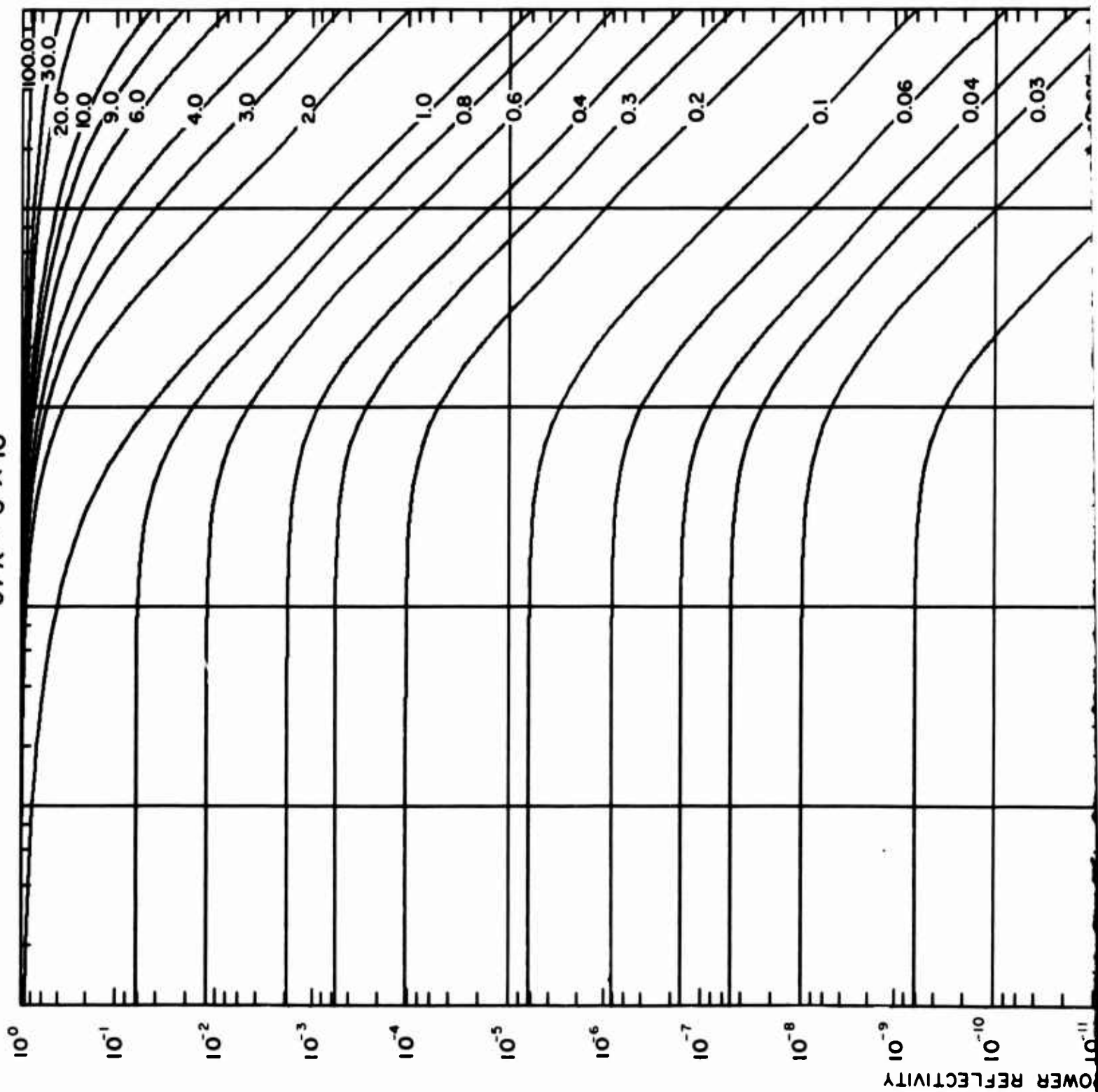


FIG. 10(a)

FIG. 10(a) POWER REFLECTION COEFFICIENT vs. Z FOR VALUES OF σ/λ INDICATED AT TOP OF FIGURE

13

$$\sigma/\lambda = 3 \times 10^{-3}$$



A

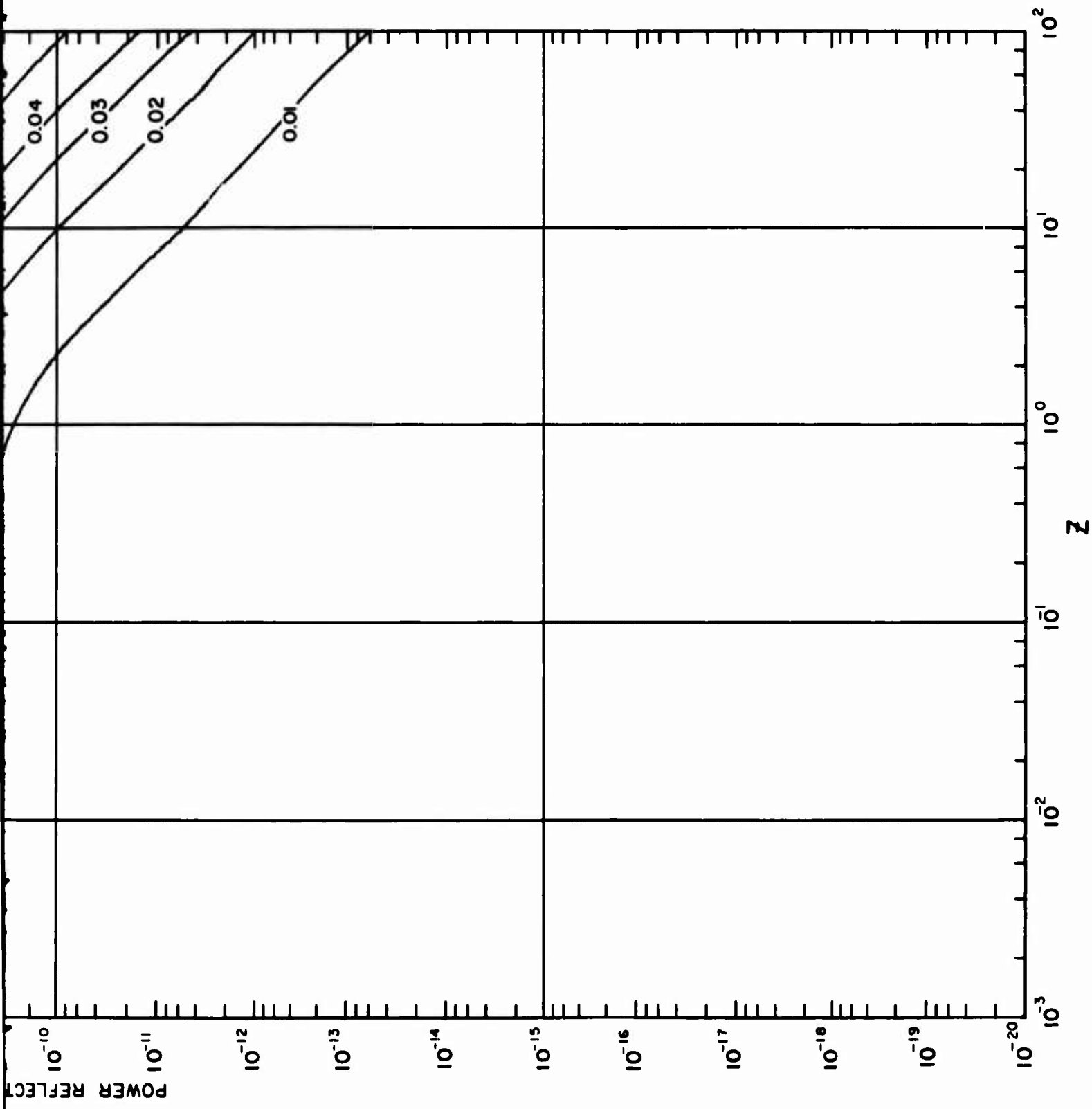
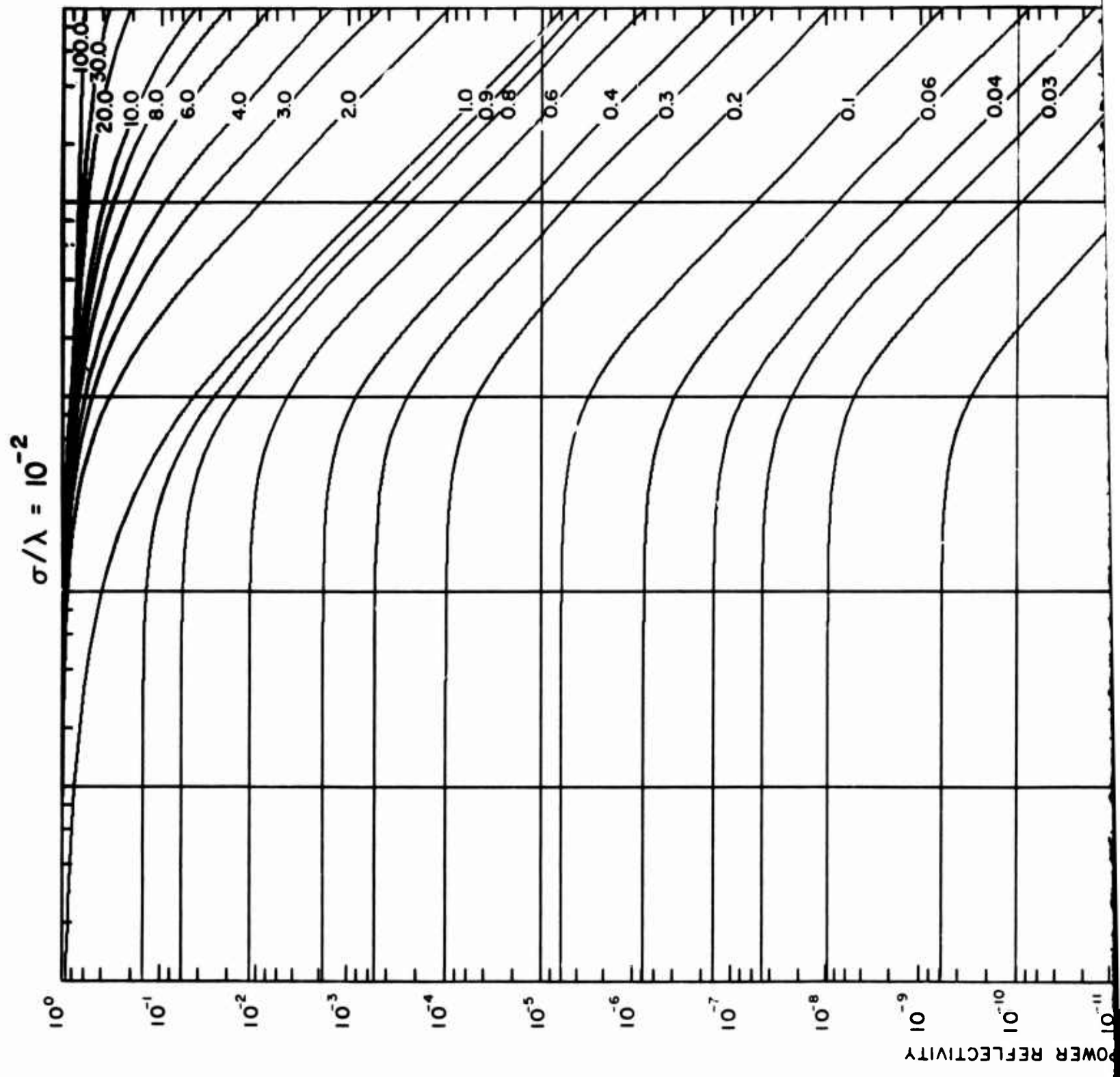


FIG. 10(b)

B

A



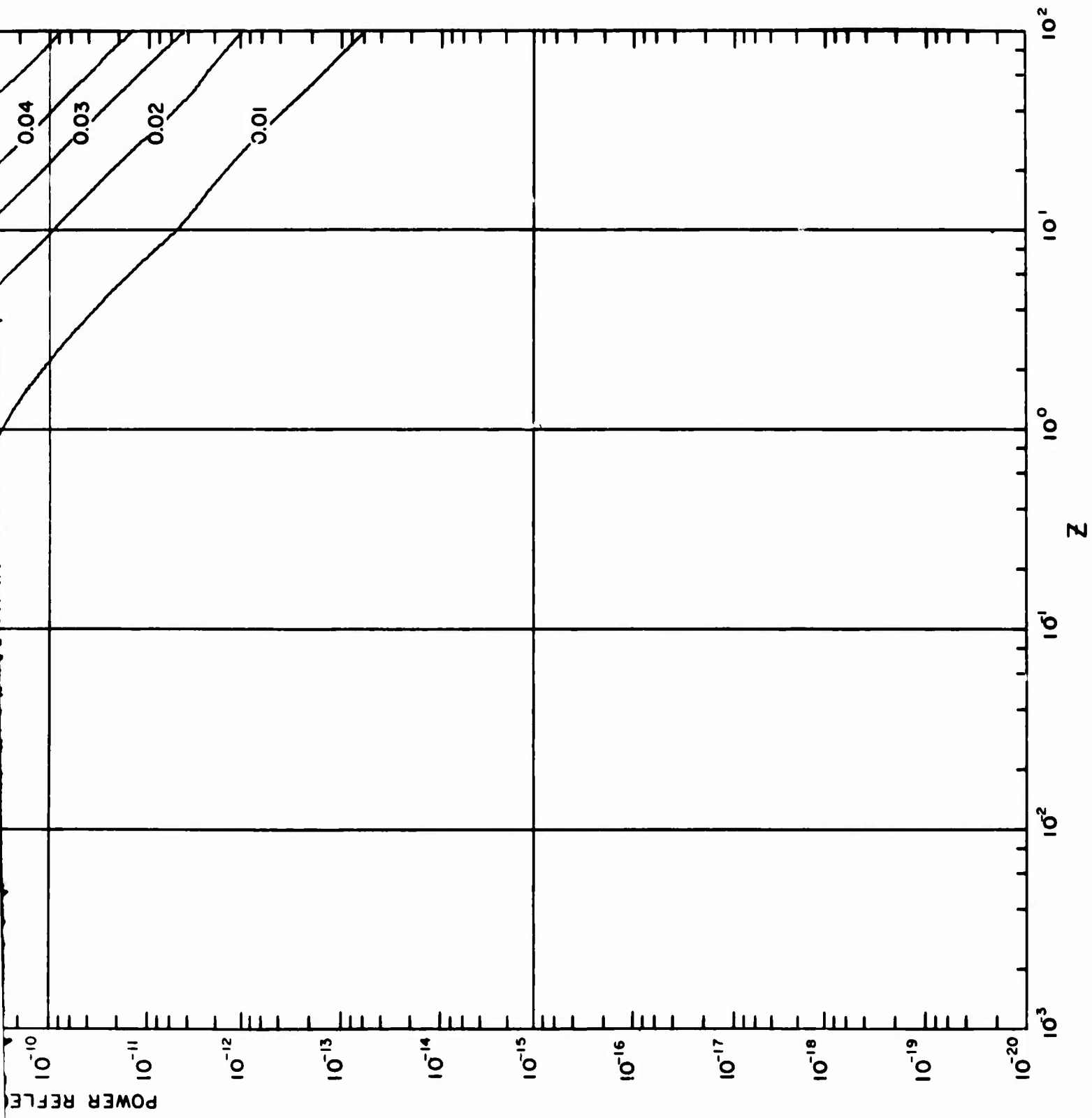
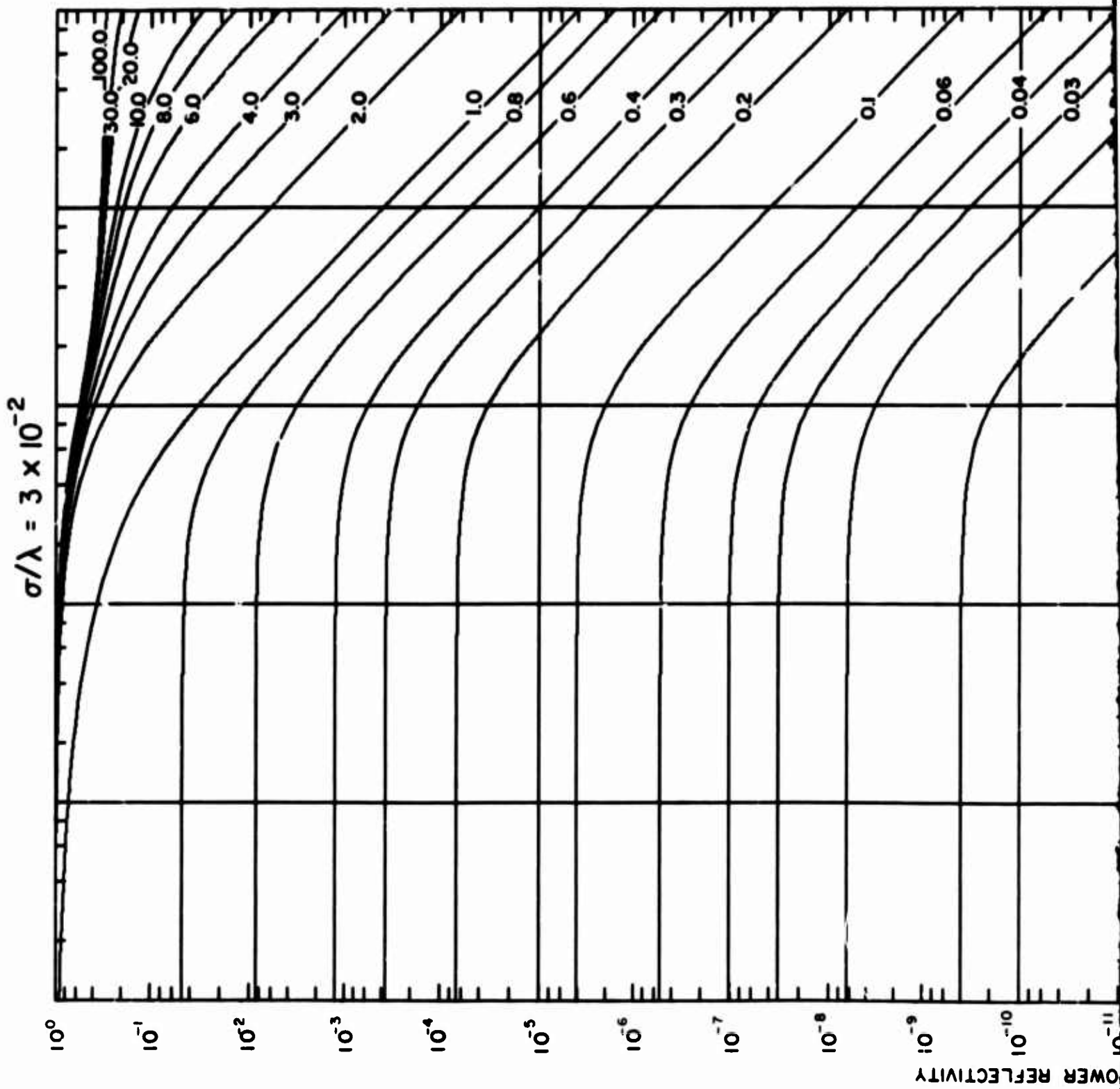


FIG. 10(c)

B

A1



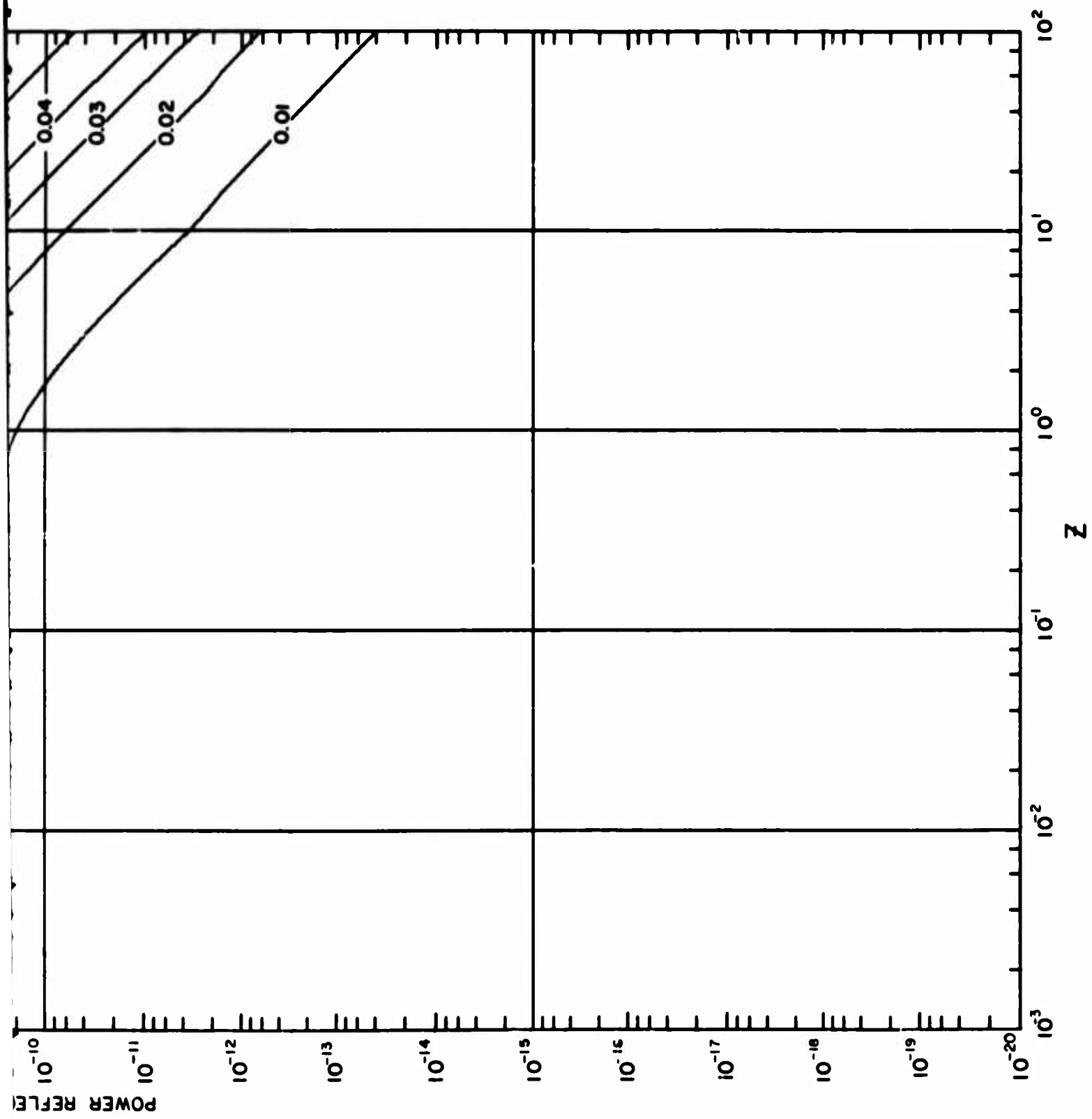
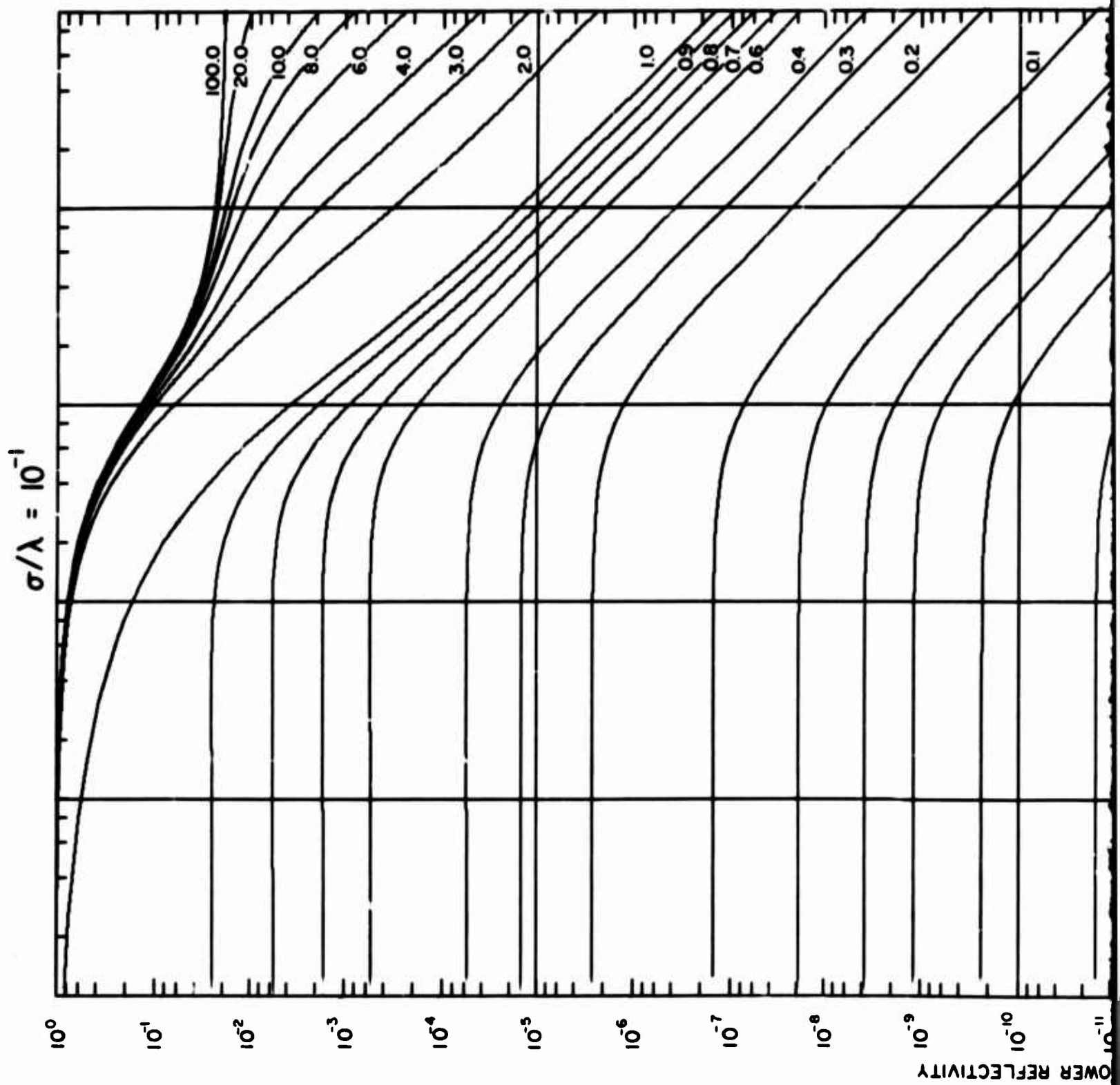


FIG. 10(d)



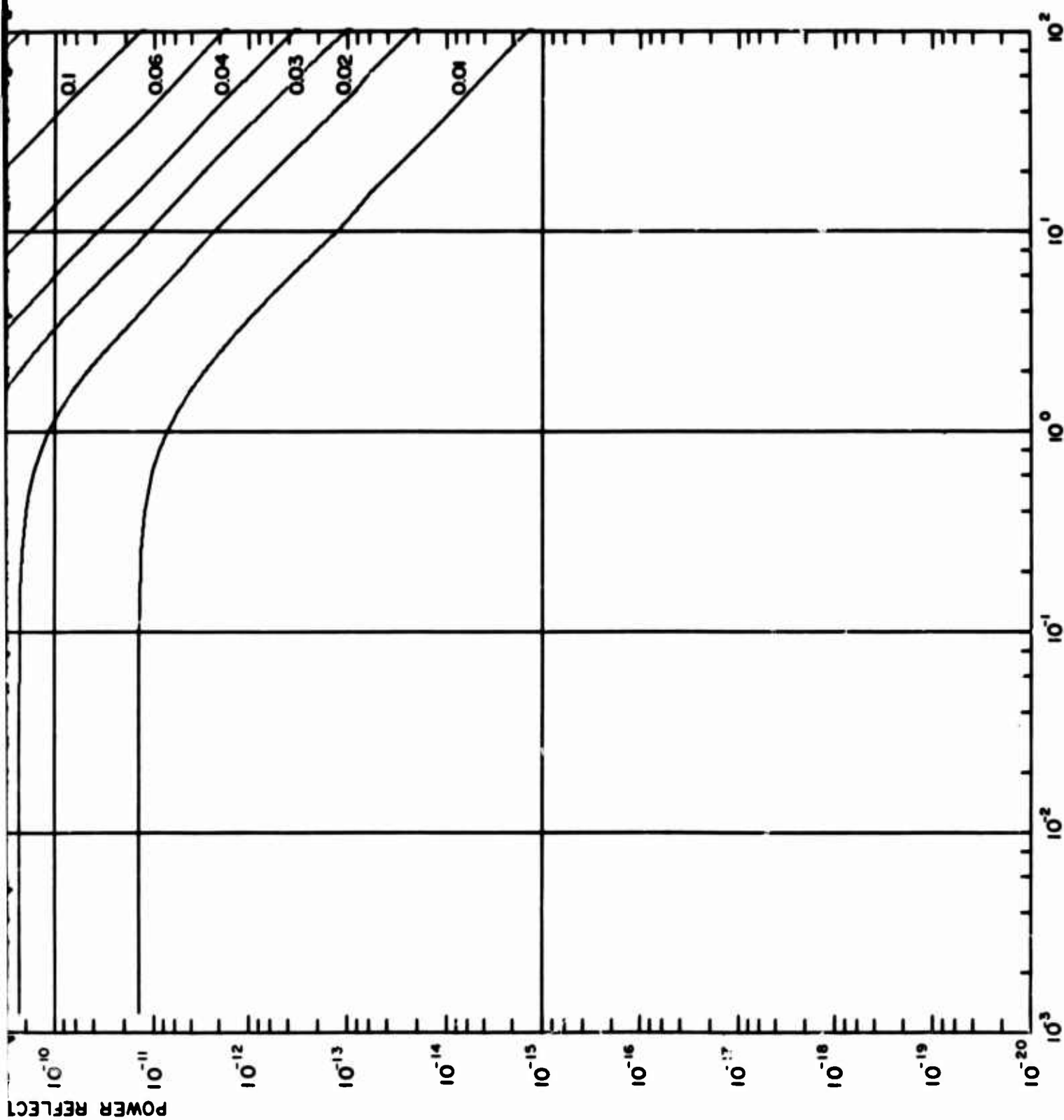
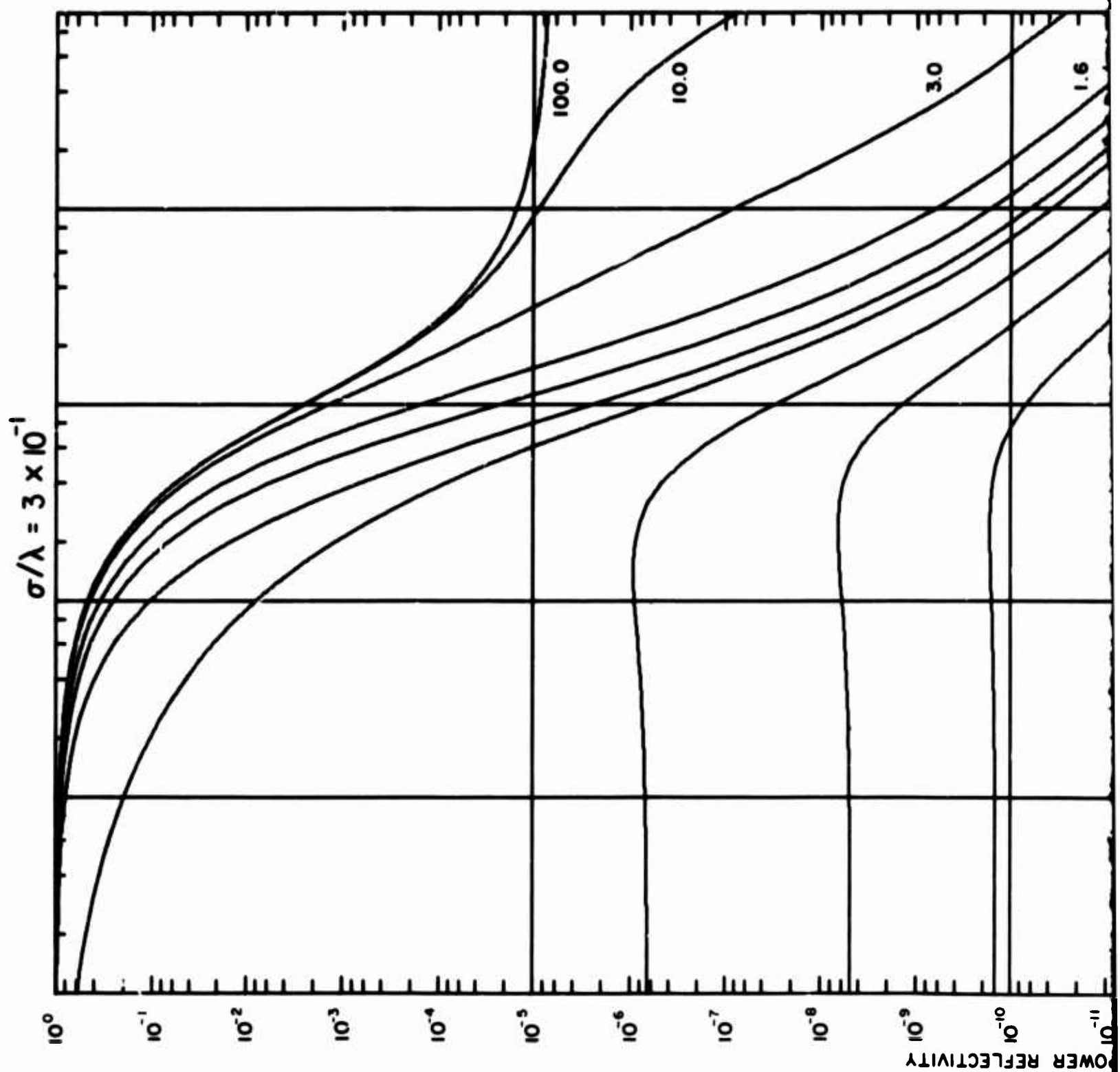


FIG. 10(e)

B

A



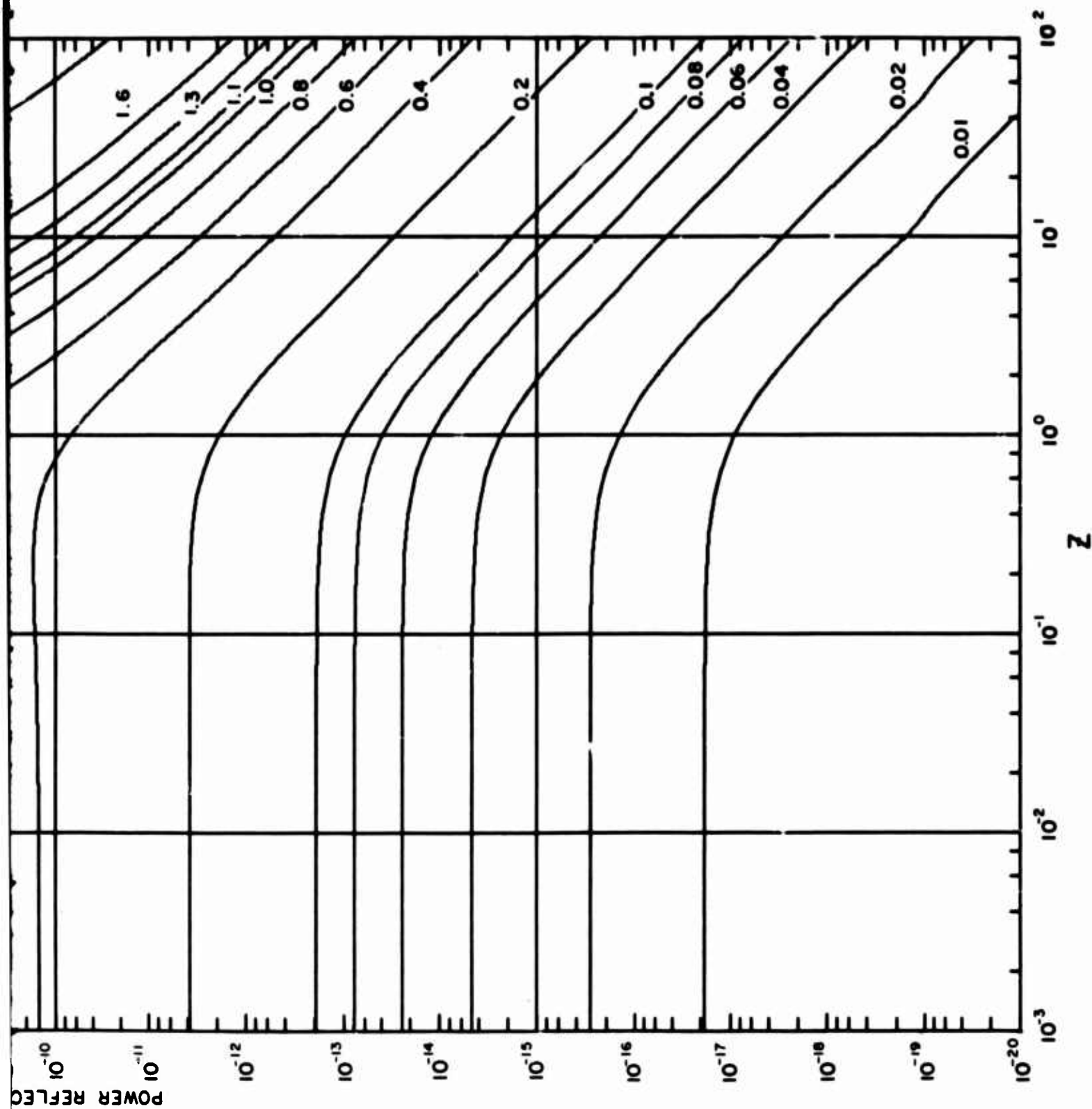
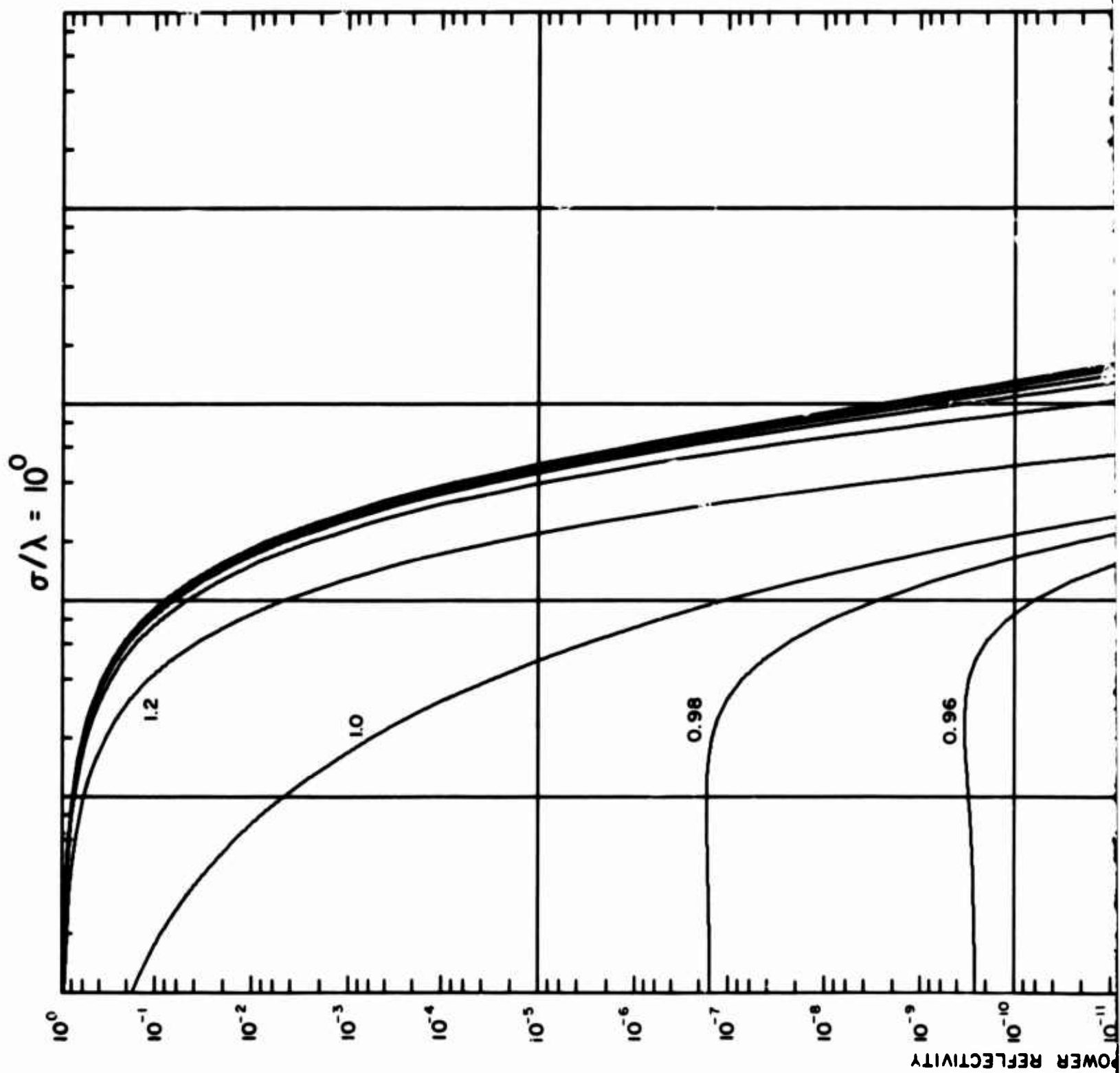


FIG. 10(n)

B



A

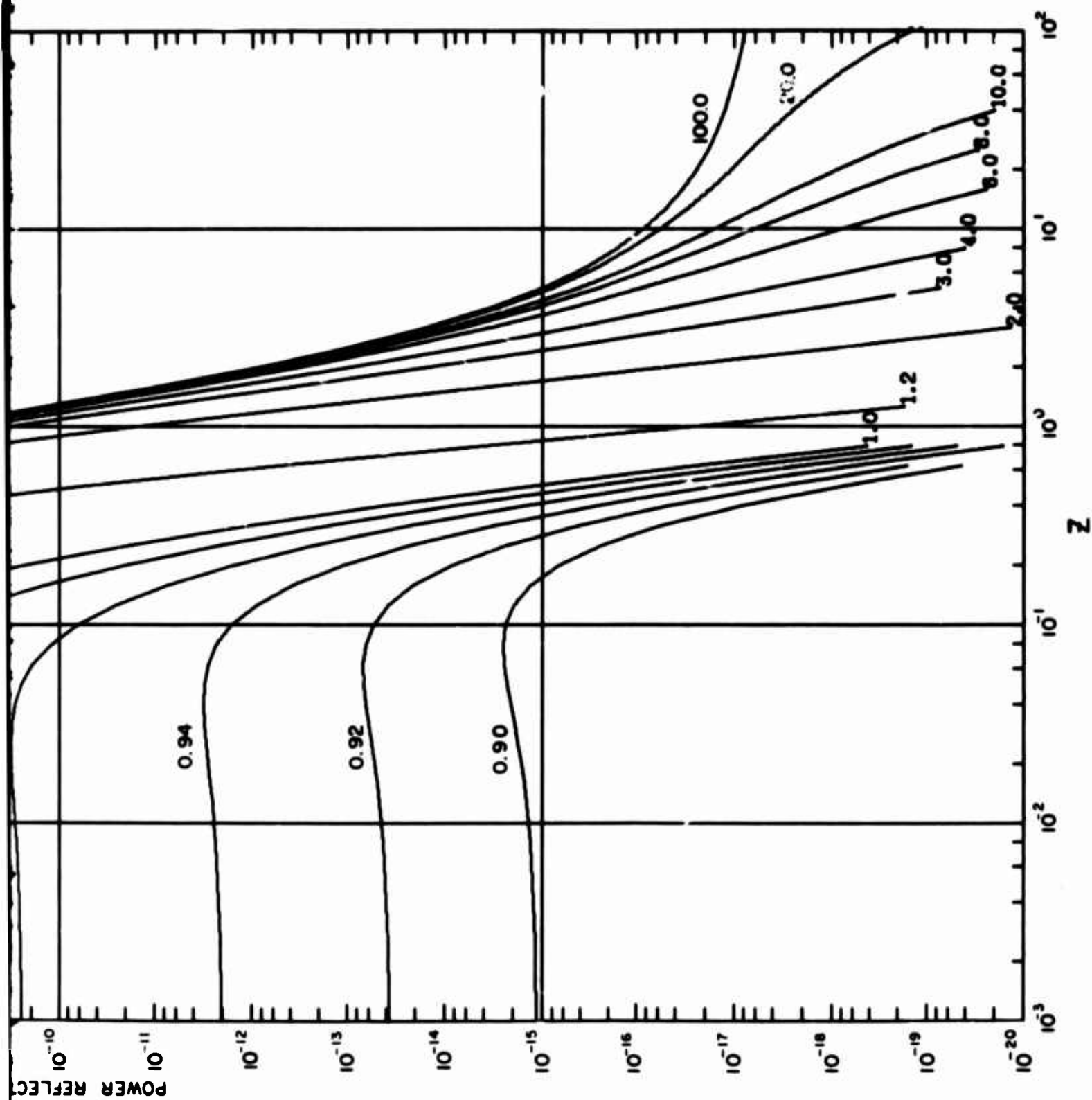
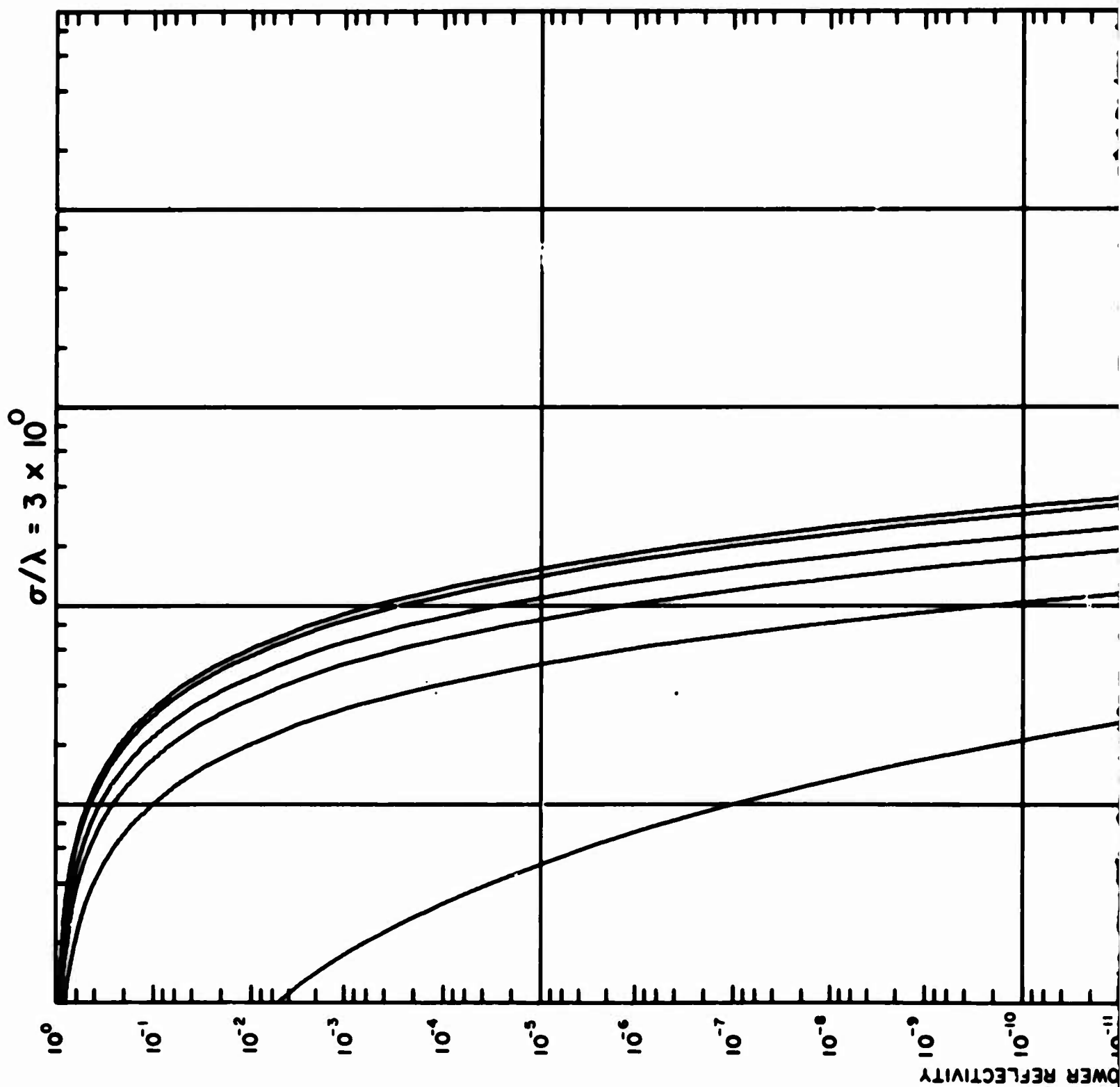


FIG. 10(a)



A

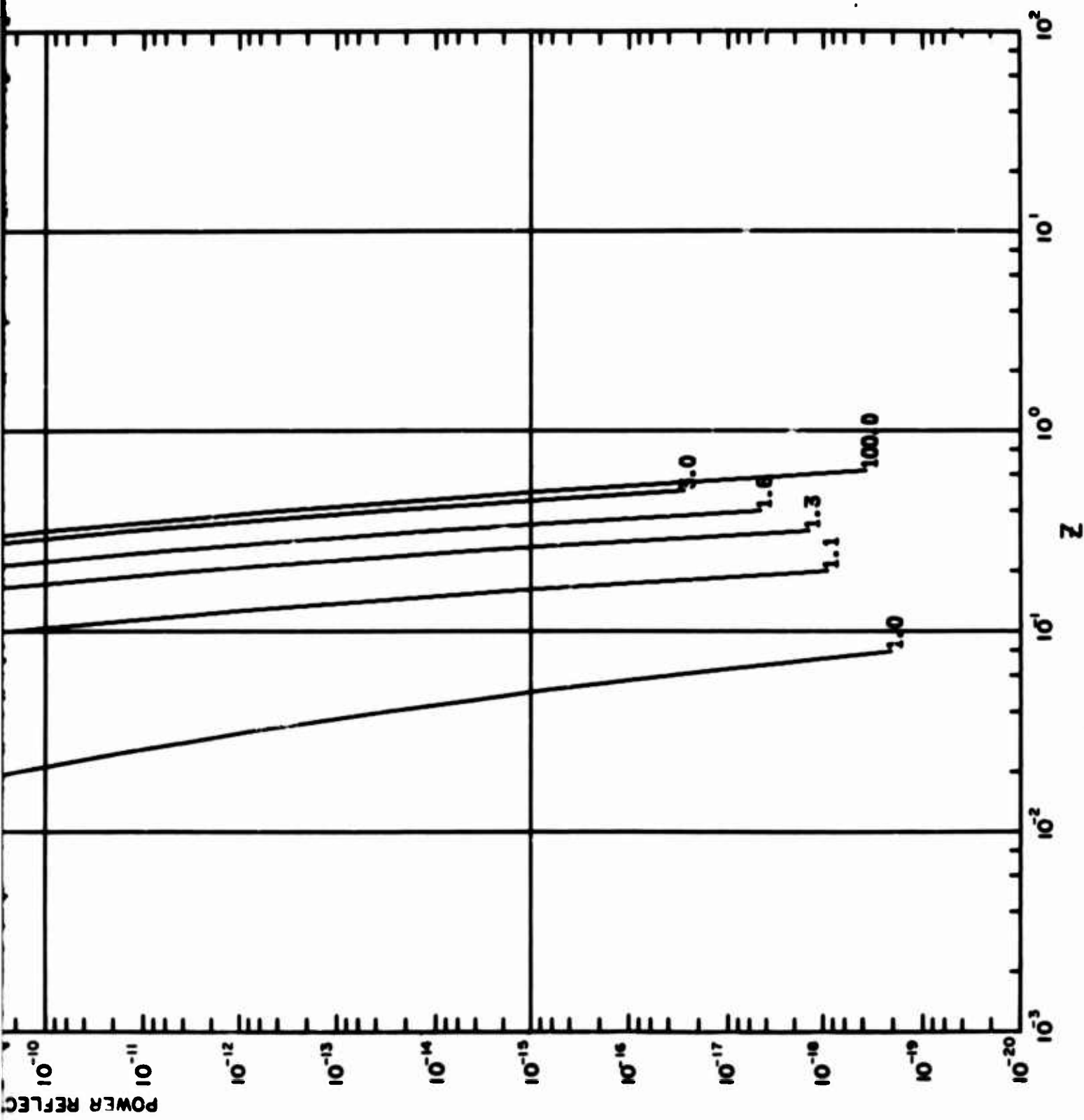
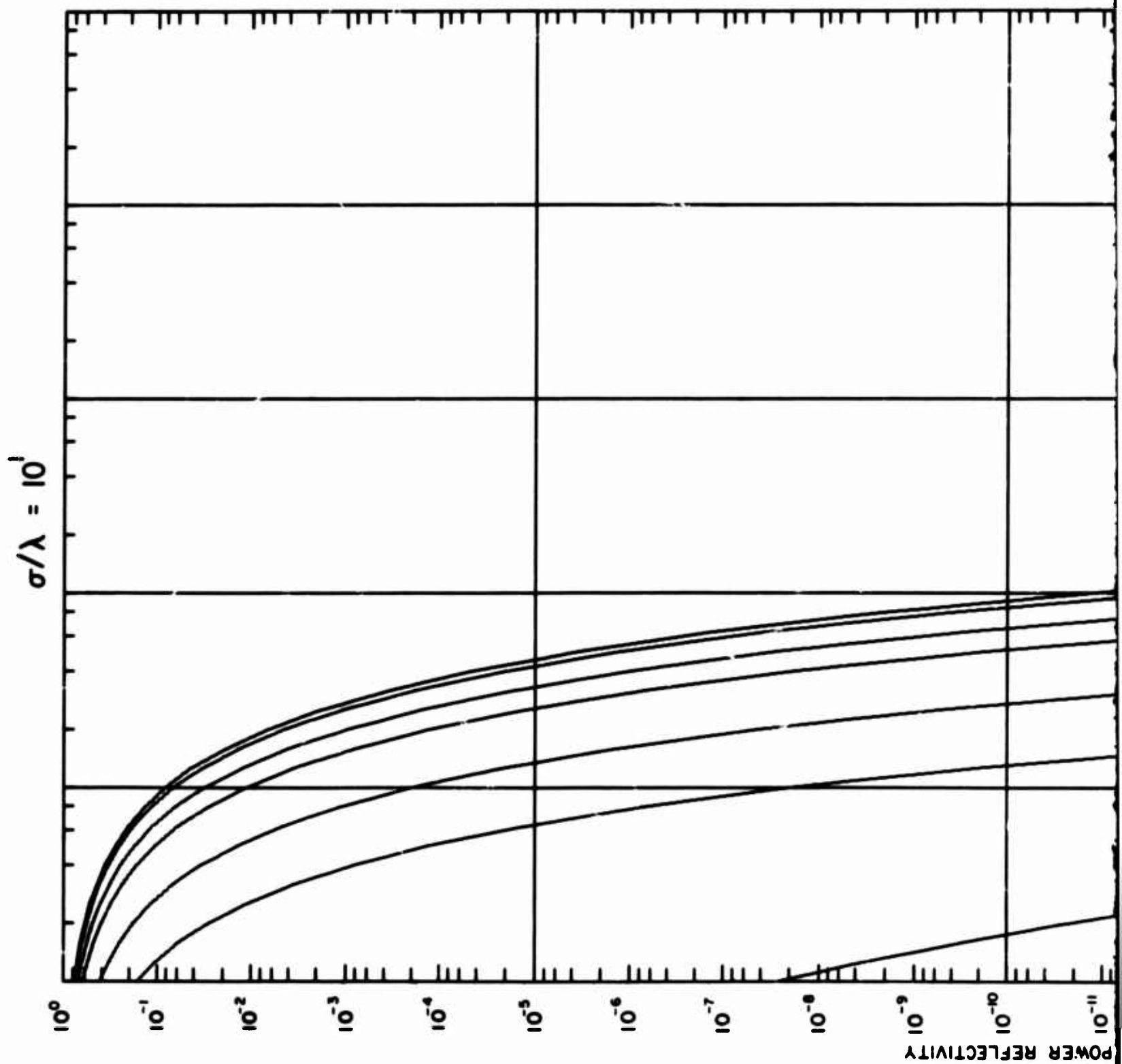


FIG. 10(n)

B



A

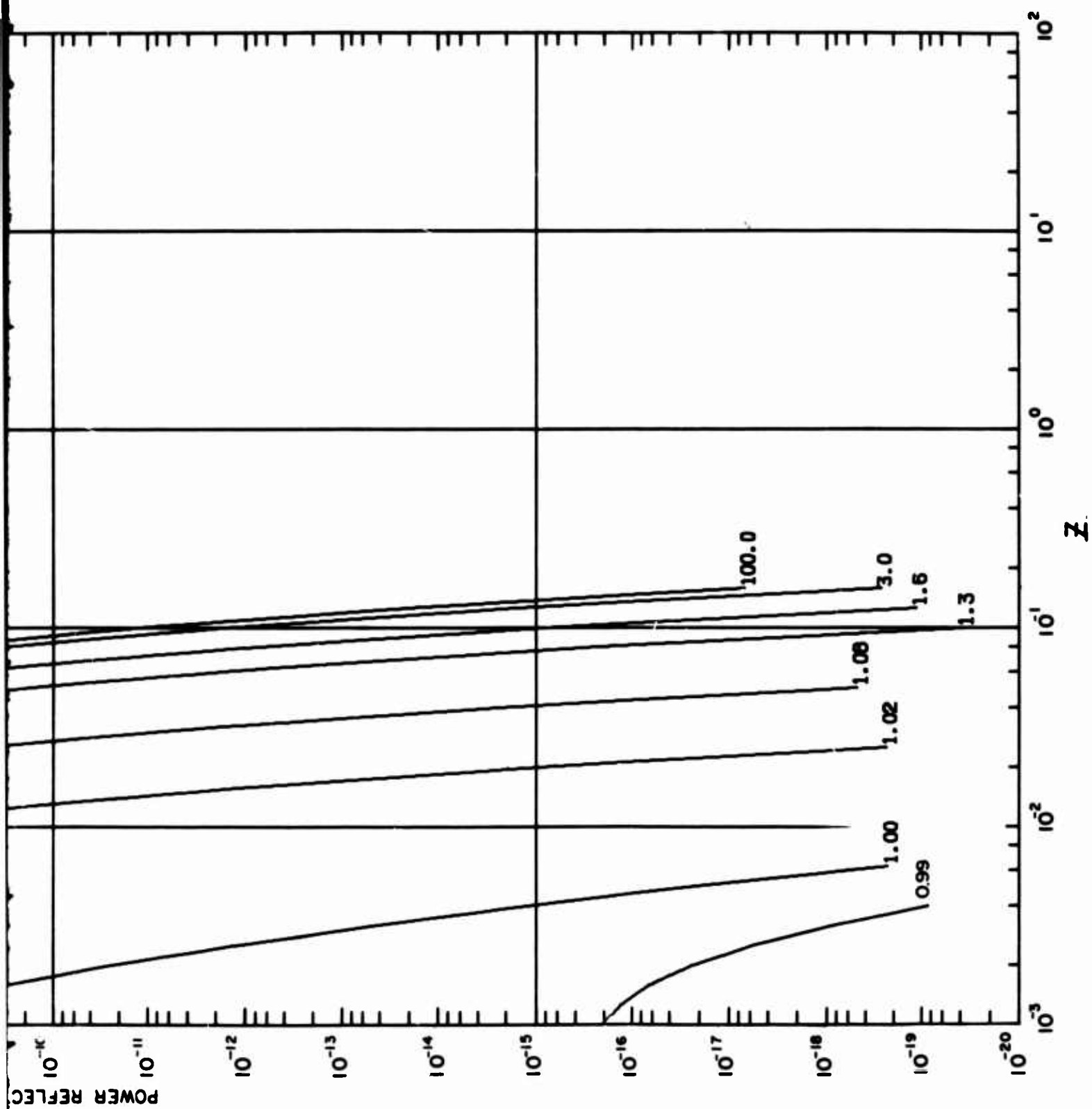
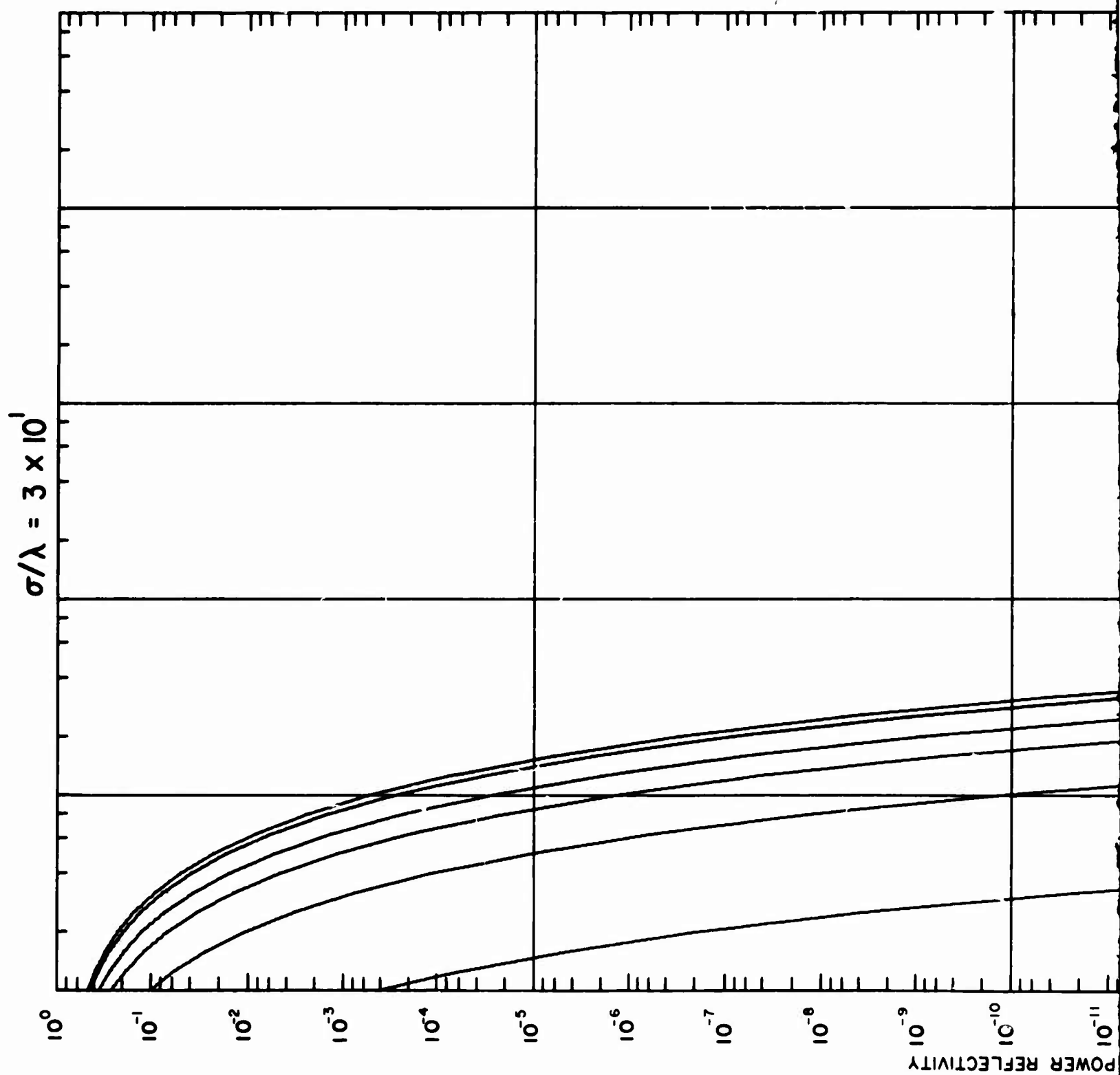


FIG. 10(ii)

B



A

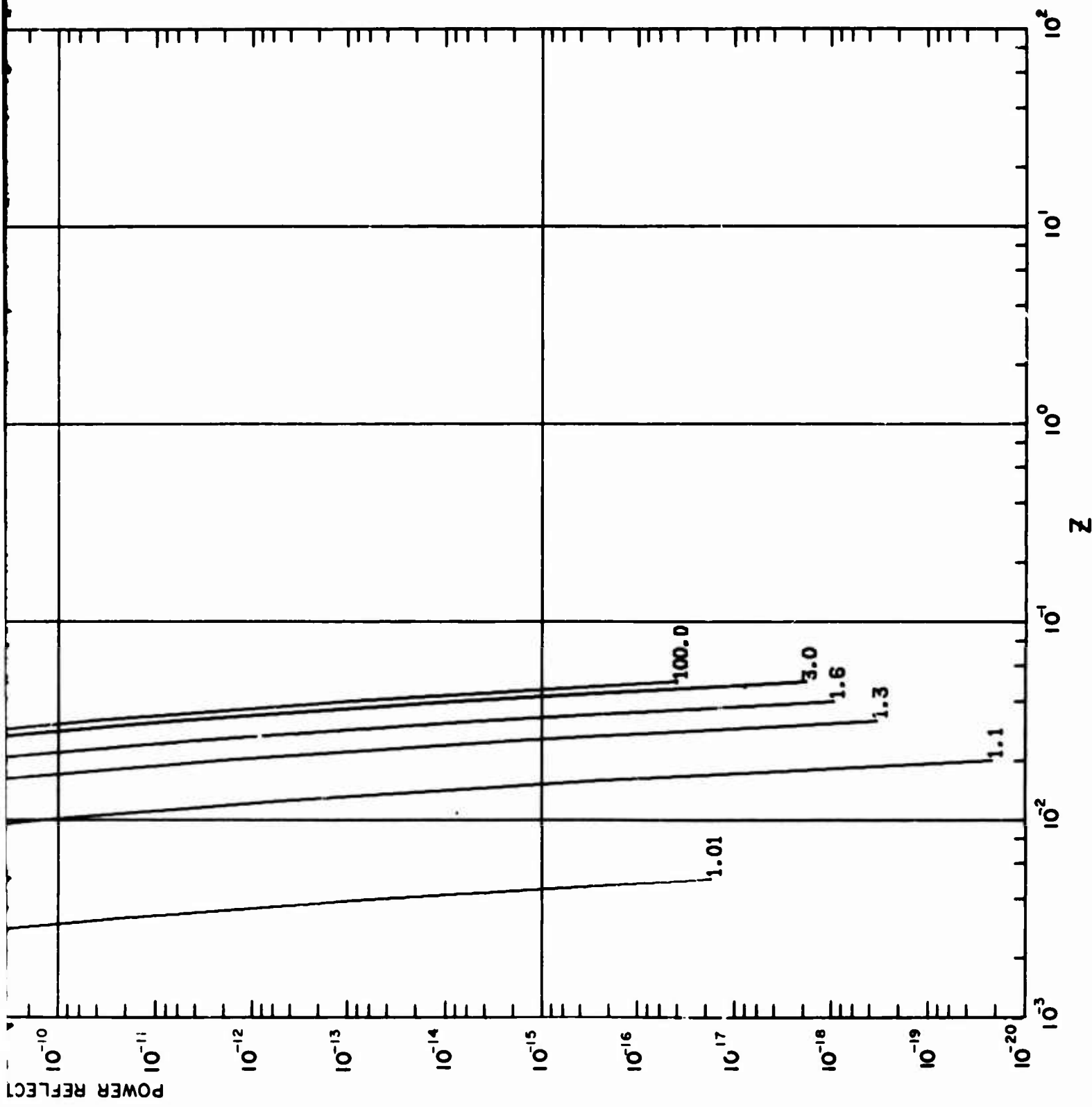
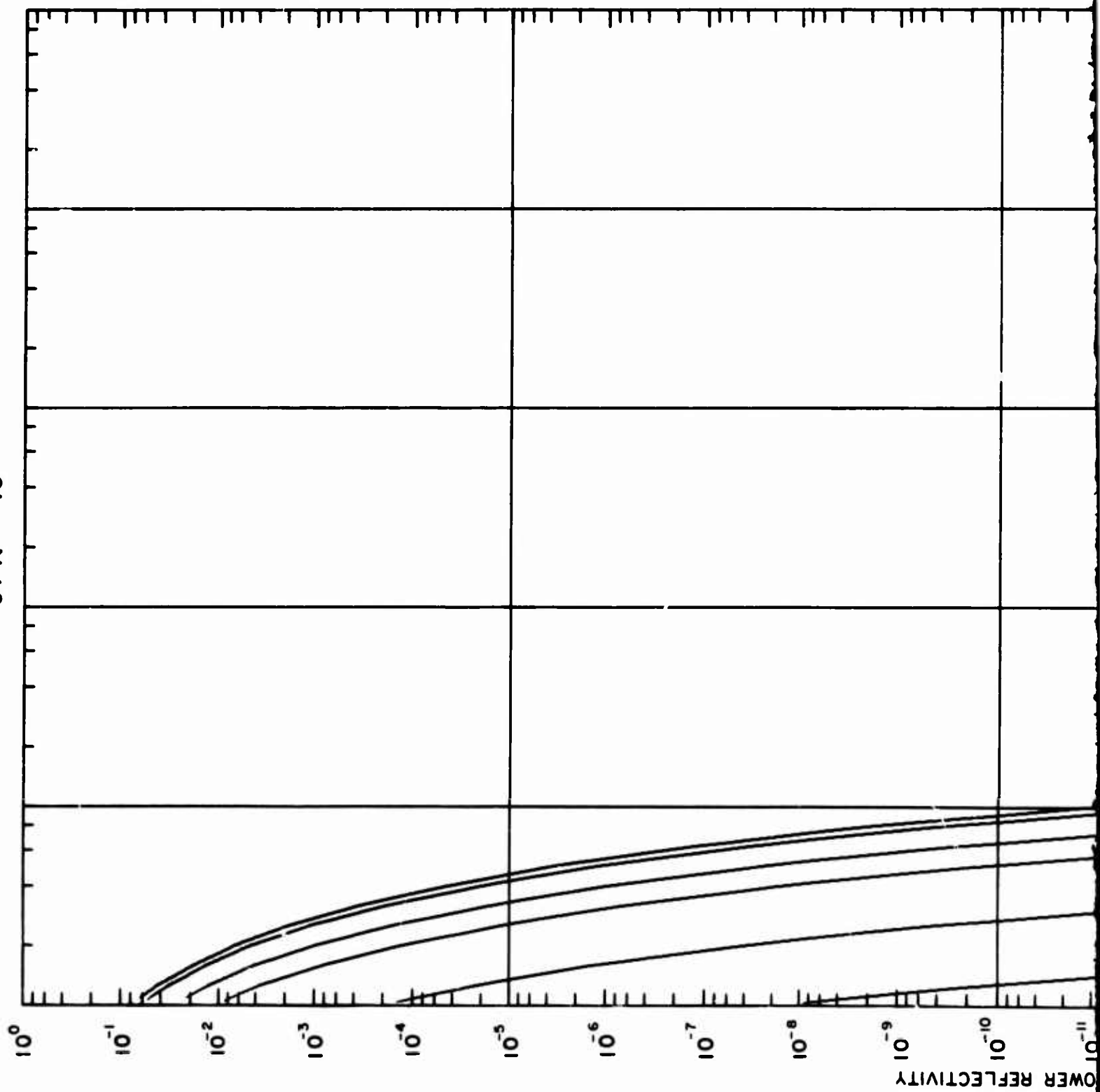


FIG. 10(j)

$$\sigma/\lambda = 10^2$$



A

B

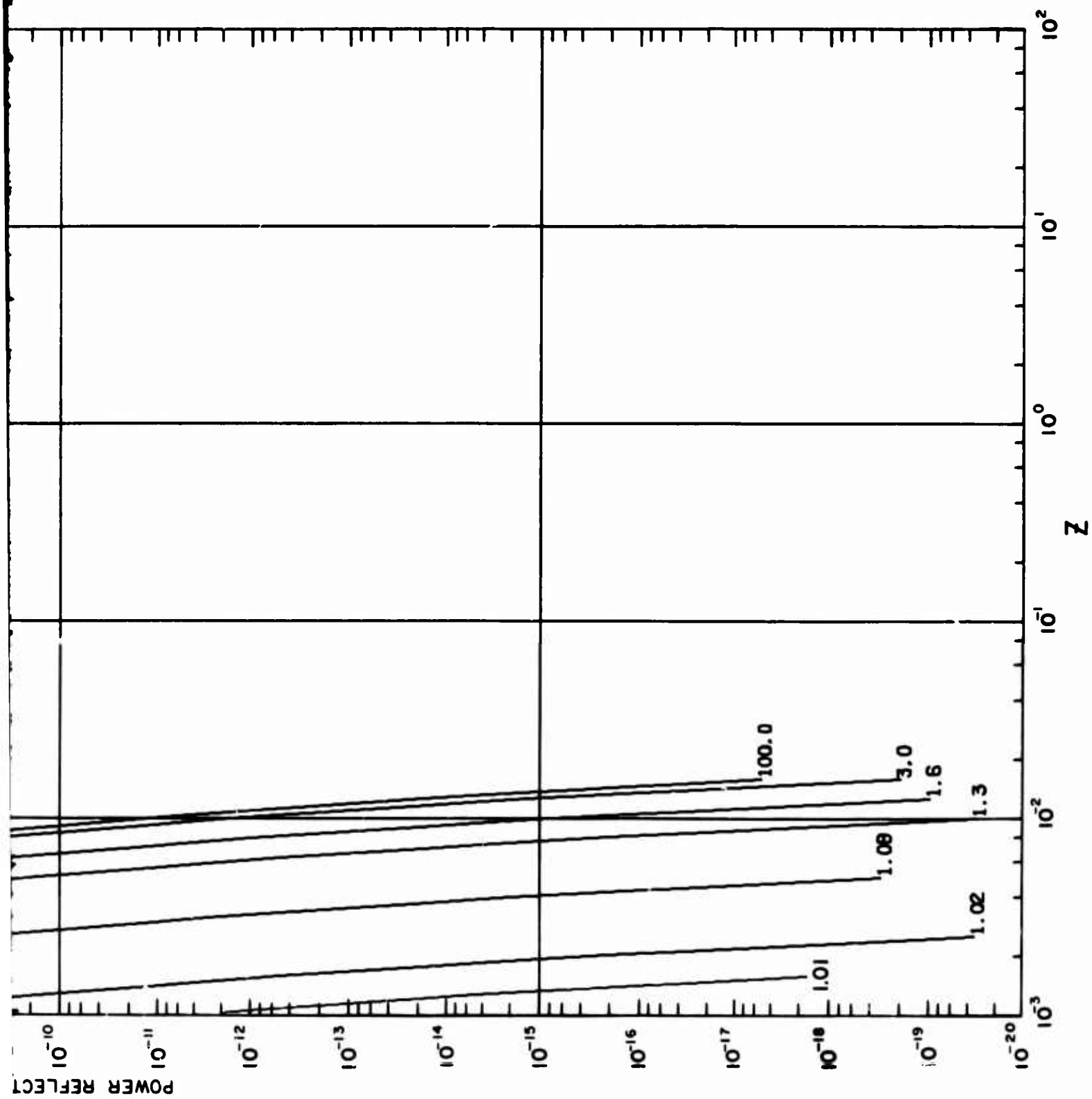
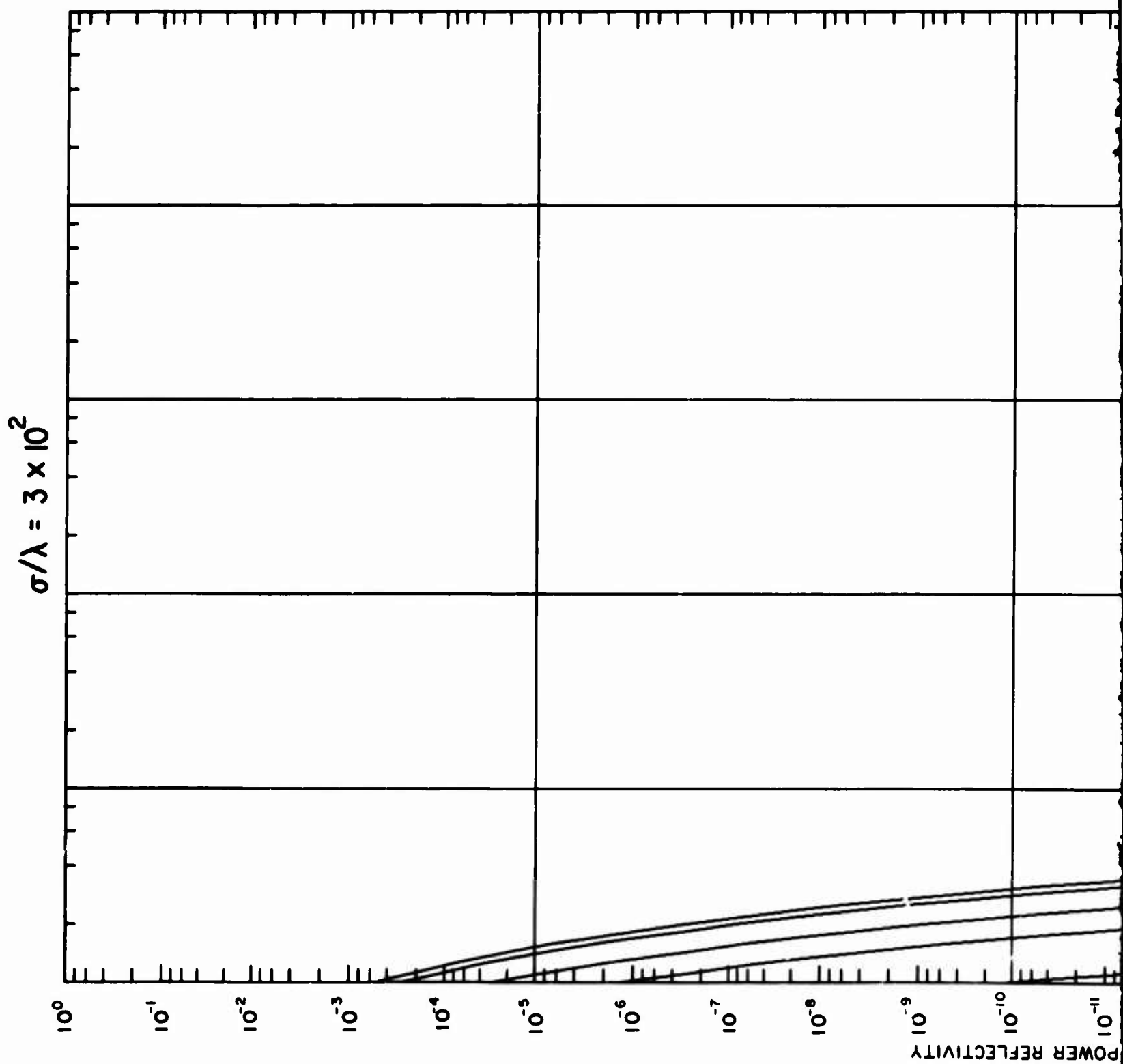


FIG. 10(k)

A



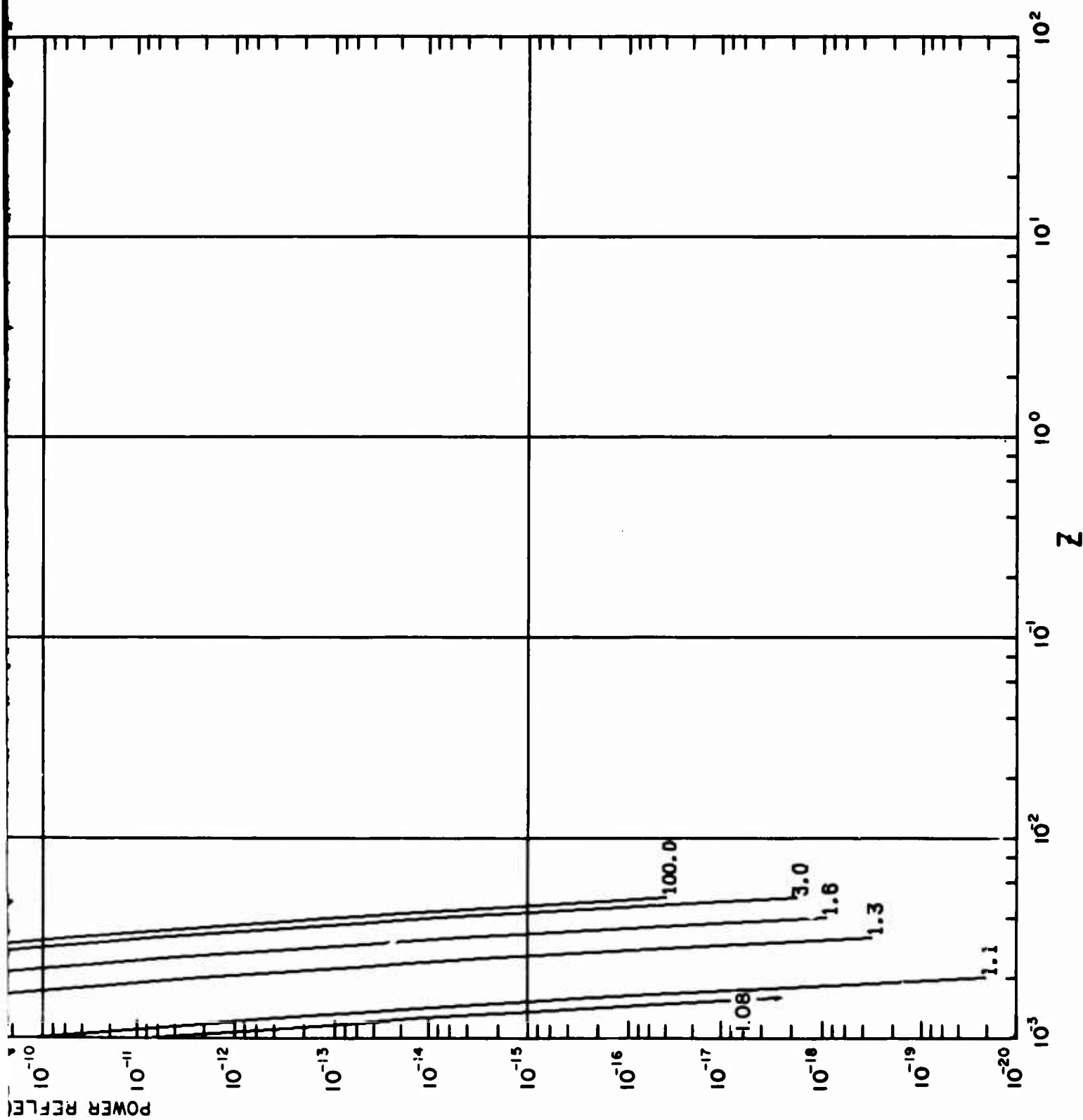
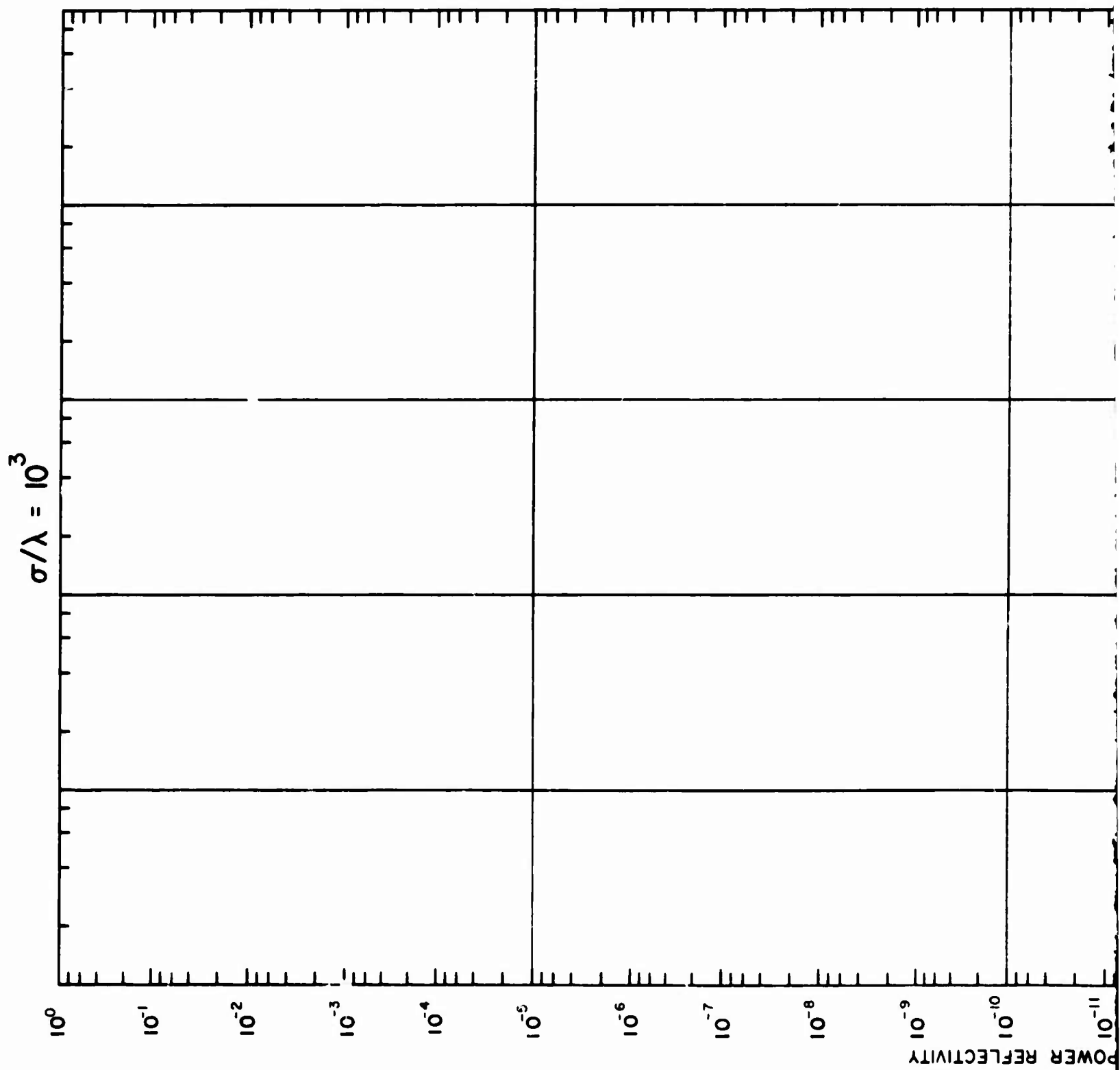


FIG. 10(I)



A

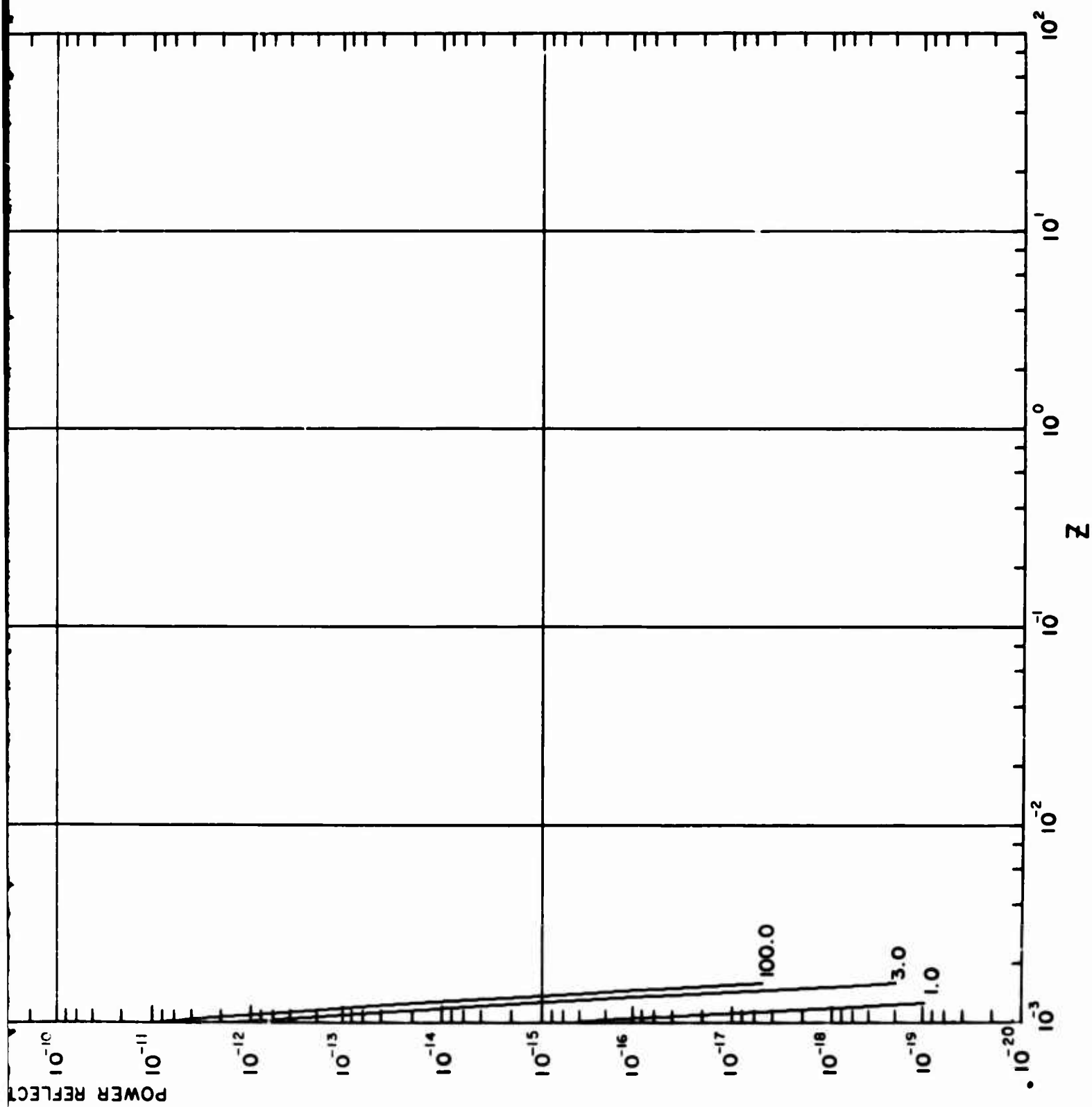


FIG. 10(m)

for large values of σ/λ aids in producing relatively unambiguous ionograms obtained with ionospheric sounding equipment. If, for instance, the ionosphere produced a very sharp interface ($\sigma/\lambda \ll 1.0$), then the power reflectivity would begin to decrease slowly with an increase in frequency above the layer critical frequency so that the observed, abrupt termination of reflection at $F_0 f_2$, for example, would not be obtained.

The second group of figures, Fig. 10(a) through 10(m), presents the power reflectivity plotted versus Z for a variety of values of σ/λ . The power reflectivity displayed in this manner demonstrates some interesting behavior. Most startling is that demonstrated in Fig. 10(g) for the situation with $\sigma/\lambda = 1.0$. For a value of $f_p/f = 0.9$, note that the power reflectivity increases very gradually as Z increases rather than decreasing as one would normally expect! The author hopes that this behavior results from a physical effect rather than from an as yet unrealized difficulty with his analytics and approximations. We note that a similar behavior occurs to a lesser extent in Fig. 10(f) for a σ/λ ratio of 3×10^{-1} .

Assuming that, indeed, the results are correct and the reflectivity does increase slightly with an increase in Z , we then believe that the slight increase results from changes in effective radar wavelength. Thus, at the proper collision frequency, the radar wavelength is "in resonance" with the plasma gradient. That is, the currents induced by the incident signal are phased just right so that their reradiation from the regions of steepest curvature in dielectric constant reinforce in the backscatter direction. At higher collision frequencies, the wavelength in the plasma is too short. At lower collision frequencies, the wavelength in plasma is too long.

For more shallow gradients (larger σ/λ ratios), f_p/f ratios would have to be larger than 0.9 in order for the effective wavelength in the plasma to be long enough to be "synchronous" with the spacing between regions of maximum dielectric curvature. In these less steep gradients, though the signal wavelengths may be nearly in resonance for f_p/f close to 1.0, absorption effects could dominate over resonance. For σ/λ ratios

less than 1.0, the spacing of regions of maximum curvature may be less than the free-space radar wavelength, so that no resonance can occur (wavelengths only increase in plasmas).

Our intuition tells us that the observed increase in reflectivity will be mathematically observable only for plasmas with σ/λ values near 1.0 and f_p/f ratios nearly equal to, but less than, 1.0. We suspect that the reflectivity values in Fig. 10(g), where this "resonance" appears, are so very low that the coherent reflection from the plasma interface phenomenon that we are studying in this report is unimportant. Therefore the resonance phenomenon is of no practical importance.

The second interesting behavior exhibited in Figs. 10(e) and 10(f) is the "plateau" in reflectivity at large values of Z and at values of f_p/f that are very large compared with 1.0. We explain this phenomenon as follows. For fixed σ/λ value, very small Z , and very large f_p/f (overdense, collisionless plasma), the reflectivity takes on the Fresnel value--which is near 1.0. The radar-wave reflection occurs primarily in the region where $f_p/f = 1.0$. As the collision frequency is increased, the reflectivity value begins to decrease as the collision-produced real part of the propagation parameter allows energy to propagate into the overdense plasma region. For very large f_p/f values, the reflectivity value will decrease as Z (the electron collision frequency) increases with values that are the same as those computed in Sec. 3.

As Z further increases, the plasma continues to appear as an exponential plasma as discussed in Sec. 3 with the arctangent Z term of Eq. (6) saturated near $\pi/2$; further increases in Z cause no observable change in arctangent Z . Thus, the reflectivity value levels off as a function of increasing Z . In the plasma, as Z is increased, the effective region where reflections occur penetrates deeper and deeper into the medium even though the power reflectivity value remains constant. Eventually, for a large enough value of Z , the wave penetrates deeply enough into the plasma so that the Eq. (8) approximation to Eq. (7) is not valid where reflection effectively takes place. That is, for the exponential plasma of Sec. 3 the region of effective reflection occurs at a depth

where the electron density exceeds that which our particular finite-density plasma achieves. Therefore, our finite-density plasma becomes ineffective as a reflector, so that further increases in Z are accompanied by a decrease in power-reflection coefficient. This intuitive explanation for the observed plateau values for reflectivity is supported by the fact that reflectivity values for the exponential plasma of Sec. 3 agrees with the plateau values. Figure 11 presents plots of reflectivity for the profile forms of Eqs. (7) and (8).

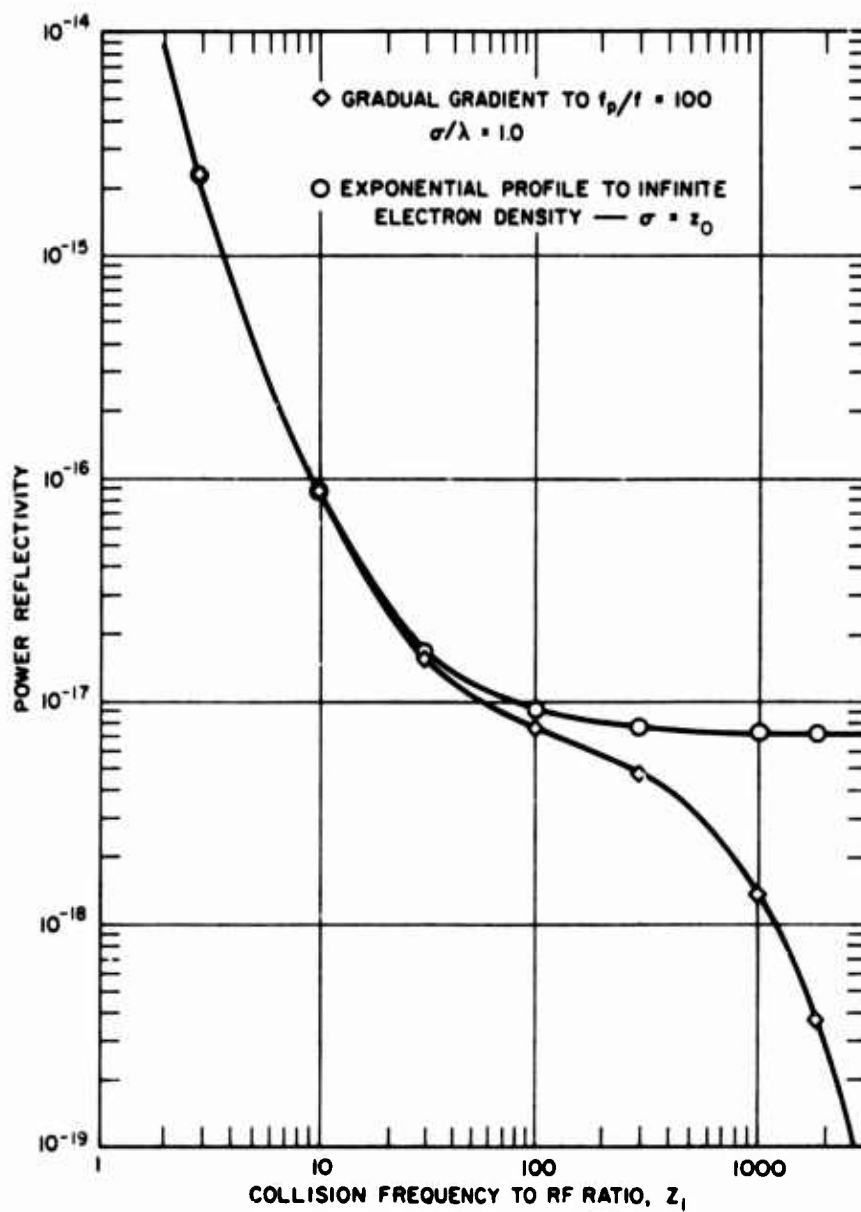


FIG. 11 COMPARISON OF POWER REFLECTIVITY vs. Z FOR A GRADUAL GRADIENT AND AN EXPONENTIAL PROFILE

5 CONCLUSIONS

In this report we have presented data to allow the reader to estimate the power reflectivity of an electromagnetic signal normally incident upon a plasma gradient. The electron collision frequency is assumed constant throughout the plasma. We have provided graphs that allow for simple determination of plasma characteristics in the atmosphere. We have presented in Sec. 3 a "slide rule" to allow easy estimation of the power reflectivity from an exponential plasma (electron density goes to infinity) in the earth's atmosphere. In Sec. 4 we have presented data for a specific analytical form for the plasma gradient to a finite, uniform electron density.

For reflection from atmospheric plasma gradients, we suggest the following procedure. Determine whether or not the atmospheric plasma is overdense to the electromagnetic signal of concern. If it is, then use the slide rule of Figs. 7 and 8 of Sec. 3 to determine the power reflectivity--provided the form of the gradient is well matched by the exponential form. If the reflectivity is large enough to be of concern, then the reader would be well advised to pursue the problem in some additional depth by means of the data presented in Sec. 4. The power reflectivity as given in Sec. 3 will always be larger. If the reflectivity is very small [less than 10^{-10} say], then, as indicated earlier in this report, some other reflection mechanism may be more important than the coherent effect of the plasma gradient. The reader would then be well advised to study other possibilities.

If the atmospheric plasma is not overdense to the signal of concern, then the data of Sec. 4 must be used. Again, the reader should be alert to the physics of his environment inasmuch as mechanisms other than smooth plasma gradients can reflect electromagnetic signals.

In either case, overdense or underdense plasmas, the reader can probably match well enough the plasma gradient in the reflecting region. Differences in plasma profiles in the exterior can be handled by comparing

signal absorption by WKB approximation for the two external regions.
Appropriate compensation can then be made.

Appendix

REDUCTION OF ANALYTIC FORM FOR REFLECTION
FROM A GRADIENT IN PLASMA DENSITY

Appendix

REDUCTION OF ANALYTIC FORM FOR REFLECTION FROM A GRADIENT IN PLASMA DENSITY

Reference 3 (Eq. 17.112) gives for the voltage reflectivity between a vacuum and a plasma the following form:

$$R_v = \frac{C - q_2}{C + q_2} \cdot \frac{(-2ik\sigma C)!}{(2ik\sigma C)!} \cdot \left[\frac{\{ik\sigma(q_2 + C)\}!}{\{ik\sigma(q_2 - C)\}!} \right]^2 \quad (A-1)$$

where

C = cosine of incidence angle ($C = 1$ for normal incidence)

$q_2^2 = \epsilon_2$ complex dielectric constant deep inside plasma

$$\epsilon_2 = 1 - \frac{X}{1 - iZ}$$

with X and Z defined in Sec. 2, and

$$k = 2\pi/\lambda$$

λ = Radar wavelength

σ = Distance characteristic of change in plasma density [see Eq. (A-2)].

The functions indicated are factorial functions that are also gamma functions. They have complex arguments.

The expression (A-1) gives the voltage reflection coefficient (including phase) from a plasma gradient in which the dielectric constant varies in the following way:

$$\epsilon(z) = 1 - \frac{X}{1 - iZ} \cdot \frac{1}{(1 + e^{-z/\sigma})} \quad (A-2)$$

This form means that the collision frequency Z is not a function of position, but X is. In fact

$$n_e(z) = n_0 \cdot \frac{1}{(1 + e^{-z/\sigma})} \quad (A-3)$$

For large negative values of z we find that the electron density is nearly a simple exponential as was used in Sec. 3. For very large negative values of z , the dielectric constant approaches the constant value 1. At very large positive values of z the electron density approaches a constant value of n_0 and the dielectric constant similarly approaches a constant value.

We shall be substituting a value 1.0 for C (we are only interested in normal incidence). The first factor on the right-hand side of Eq. (A-1) we call the Fresnel term. This factor gives the plasma reflection coefficient should the plasma gradient be exceedingly abrupt compared with the wavelength.

The second factor on the right-hand side of Eq. (A-1) has modulus 1.0 and only affects the phase of the returning signal. We shall be concerned only with power reflectivity, so that this factor is of no concern. However, as an exercise, for real σ (the only kind we consider),

$$\left| \frac{(-2ik\sigma)!}{(+2ik\sigma)!} \right|^2 = \left| \frac{\Gamma(1-2ik\sigma)}{\Gamma(1+2ik\sigma)} \right|^2 = \left| \frac{\Gamma(\bar{z})}{\Gamma(z)} \right|^2 \quad (A-4)$$

with z here meaning the complex number $(1 + 2ik\sigma)$. If we apply the identity

$$\Gamma(z) = \overline{\Gamma(\bar{z})} \quad (A-5)$$

we find

$$\left| \frac{\Gamma(\bar{z})}{\Gamma(z)} \right|^2 = \frac{\overline{\Gamma(\bar{z})} \Gamma(\bar{z})}{\Gamma(z) \Gamma(z)} = \frac{\Gamma(z) \Gamma(\bar{z})}{\Gamma(\bar{z}) \Gamma(z)} = 1.0 \quad (A-6)$$

An evaluation of the third factor on the right-hand side is not straightforward. We find

$$\left| \frac{[\{ik\sigma(q_2 + 1)\}]!}{[\{ik\sigma(q_2 - 1)\}]!} \right|^2 = \left| \frac{[\Gamma[1 + ik\sigma(q_2 + 1)]]}{[\Gamma[1 + ik\sigma(q_2 - 1)]]} \right|^2 \quad (A-7)$$

We let

$$a_2 = \mu - i\chi \quad (A-8)$$

and find

$$\left| \frac{\{\Gamma[1 + k\sigma\chi + ik\sigma(\mu + 1)]\}^2}{\{\Gamma[1 + k\sigma\chi + ik\sigma(\mu - 1)]\}^2} \right|^2 = \left| \frac{[\Gamma(x + iy_1)]^2}{[\Gamma(x + iy_2)]^2} \right|^2 \quad (A-9)$$

with

$$x = 1 + k\sigma\chi \quad (A-10)$$

$$y_1 = k\sigma(\mu + 1) \quad (A-11)$$

$$y_2 = k\sigma(\mu - 1) \quad (A-12)$$

Reference 4 [Eq. (6.1.25), p. 256] allows us the following transformations:

$$\left\{ \frac{|\Gamma(x + iy_1)|^2}{|\Gamma(x)|^2} \right\}^2 = \left\{ \frac{\prod_{n=0}^{\infty} \frac{1}{1 + \frac{y_1^2}{(x+n)^2}}}{\prod_{n=0}^{\infty} \frac{1}{1 + \frac{y_2^2}{(x+n)^2}}} \right\}^2 \quad (A-13)$$

or

$$\frac{\prod_{n=0}^{\infty} \left\{ 1 + \frac{[k\sigma(\mu - 1)]^2}{[n + 1 + k\sigma\chi]^2} \right\}^2}{\prod_{n=0}^{\infty} \left\{ 1 + \frac{[k\sigma(\mu + 1)]^2}{[n + 1 + k\sigma\chi]^2} \right\}^2} = \prod_{m=1}^{\infty} \left\{ \frac{1 + \left[\frac{k\sigma(\mu - 1)}{m + k\sigma\chi} \right]^2}{1 + \left[\frac{k\sigma(\mu + 1)}{m + k\sigma\chi} \right]^2} \right\}^2 \quad (A-14)$$

We have found that the evaluation of the continued product of Eq. (A-14) may require tens of thousands of factors before the continued product converges to within a few percent of the proper answer. We

later develop a technique that combines continued product evaluation and analytical approximation to obtain an accurate representation of Eq. (A-14) using a very reasonable amount of computer time. We return to this shortly.

The determination of μ and χ from q_2^2 is fairly straightforward. We find that by defining

$$\epsilon_r = 1 - \frac{X}{1 + Z^2} \quad (A-15)$$

and

$$\epsilon_i = \frac{XZ}{1 + Z^2}, \quad (A-16)$$

$$\mu = + \left\{ \frac{\sqrt{\epsilon_r^2 + \epsilon_i^2} + \epsilon_r}{2} \right\}^{\frac{1}{2}}, \quad (A-17)$$

and

$$\chi = + \left\{ \frac{\sqrt{\epsilon_r^2 + \epsilon_i^2} - \epsilon_r}{2} \right\}^{\frac{1}{2}}. \quad (A-18)$$

In terms of μ and χ , the first factor on the right-hand side of Eq. (A-1), when absolute squared, becomes the very familiar

$$R_F = \frac{(1 - \mu)^2 + \chi^2}{(1 + \mu)^2 + \chi^2} \quad (A-19)$$

which is the power-reflection coefficient for an abrupt interface between the plasma and a vacuum. This is known as the Fresnel equation and we call it, here, the Fresnel term.

We see from Eq. (A-1) and (A-14) that should the parameter γ approach zero (an abrupt interface), then, since Eq. (A-14) takes the value 1.0, Eq. (A-1) does reduce to the expected form.

Because of the slow convergence of Eq. (A-14) we have developed an analytic approximation to evaluate the continued product. In our computations we have usually used a hybrid form, as will be described. We define

$$a = k\sigma(\mu - 1) \quad (A-20)$$

$$b = k\sigma(\mu + 1) \quad (A-21)$$

and

$$y_m = m + k\sigma\chi \quad (A-22)$$

Then Eq. (A-14) becomes

$$P = \prod_{m=1}^{\infty} \left\{ \frac{y_m^2 + a^2}{y_m^2 + b^2} \right\}^2 \quad (A-23)$$

or

$$P = \exp \left\{ \log_e \prod_{m=1}^{\infty} []^2 \right\} = \exp \left\{ 2 \sum_{m=1}^{\infty} \log [] \right\} \quad (A-24)$$

or

$$P = \exp \left\{ 2 \left[\sum_{m=1}^{\infty} \log_e (y_m^2 + a^2) - \sum_{m=1}^{\infty} \log_e (y_m^2 + b^2) \right] \right\} \quad (A-25)$$

Transforming the summations to integrals we find

$$P = \exp \left\{ 2 \left[\int_{m=1/2}^{\infty} \log [y_m^2 + a^2] dm - \int_{m=1/2}^{\infty} \log [y_m^2 + b^2] dm \right] \right\} \quad (A-26)$$

The integrals may be evaluated to give

$$P = \exp \left\{ 2 \left[\pi(a - b) - y_m \log_e \left[\frac{y_m^2 + a^2}{y_m^2 + b^2} \right] + 2 \left[b \tan^{-1} \frac{y_m}{b} - a \tan^{-1} \frac{y_m}{a} \right] \right] \right\} . \quad (A-27)$$

We have freedom in the way we may use this analytical expression in that y_m , the lower limit, can be chosen at will. Thus, if we wish to use Eq. (A-27) as a complete substitute for the continued product, then

$$y_m = \frac{1}{2} + k\sigma\chi \quad . \quad (A-28)$$

We use the $1/2$ in Eq. (A-28) since the value for $m = 1$ of the continued product is approximated by the integral from $m = 1/2$ to $m = 3/2$. Test cases run on computers have shown that Eq. (A-27) may be in error as a complete representation of the continued product of Eq. (A-14) by as much as 10 percent, but usually, particularly for total reflectivities larger than 10^{-20} , has an accuracy exceeding one percent. Greater accuracy can be obtained by evaluating the first m factors of the continued product, and then multiplying these by Eq. (A-27) with

$$y_m = m + \frac{1}{2} + k\sigma\chi \quad . \quad (A-29)$$

Table A-1 presents a matrix of values for power reflectivity in which the hybrid form of m factors of continued product times the analytic form from $m + 1/2$ to ∞ was evaluated. From the table we see that the analytic approximation always gives one percent accuracy for reflectivities larger than 10^{-20} .

Evaluation of the continued product by itself more often than not required thousands of factors to converge to within 10 percent of the proper limit. Table A-1 also presents some results on this aspect. For all plots presented in Sec. 4 of this report, a hybrid calculation combining the first 10 factors times the analytic expression from $(10 + 1/2)$

Table A-1
TEST CASES TO EVALUATE ACCURACY OF HYBRID ANALYTIC
APPROXIMATION TO CONTINUED PRODUCT*

$\frac{Z}{X}$	0.1	10^1	10^3
0.1	3.12 - 32 1,2,10;3.0	2.95 - 36 1,3,10;3.2	2.00 - 40 2,3,10;3.2
1.0	1.15 - 07 0,0,0;0.7	8.41 - 23 1,3,10;3.2	2.08 - 37 2,3,10;3.2
10	6.80 - 02 0,0,0;0.5	2.58 - 23 0,0,10;3.4	2.93 - 36 1,3,10;3.2
10^2	8.03 - 02 0,0,0;1.6	1.61 - 17 0,0,0;7.3	7.10 - 33 1,3,10;3.2
10^3	8.15 - 02 0,0,0;5.0	7.40 - 17 0,0,0;23.5	4.86 - 24 0,0,7;3.5

*For these calculations, $k\sigma = 2\pi\sigma/\lambda = 2\pi$; $\sigma = \lambda$. Each cell of the table presents the following information: Power reflectivity (floating point representation), number of factors of continued product times analytic expression to give respectively 10 percent accuracy, 5 percent accuracy, and 1 percent accuracy; percent error when 10,000 terms of the continued product are used without the analytic extension to infinity. Example: $X = 0.1$, $Z = 0.1$; reflectivity = 3.12×10^{-32} . One term of continued product times analytic expression from $(1 + 1/2)$ to ∞ gave 10 percent accuracy, etc., and using only the continued product expression to 10,000 factors, the answer was still 3 percent in error. A zero means only the analytic approximation was needed.

to infinity was used. We assume that all points are then accurate to within one percent.

The form of the dielectric constant used in Sec. 3 is a special form of that used in Sec. 4. The analytic form of Sec. 3 is obtained if in Eq. (A-2) we set $\sigma = z_0$ and let X go to infinity. As a check on the validity of our procedures, we used our analytic approximation to the continued product, allowed X to approach infinity and, indeed, found convergence to the expression given in Eq. (6) of Sec. 3. The procedure is tedious and will not be reproduced here.

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13. ABSTRACT <p>This report presents numerical results in the form of graphs of the power reflection coefficient for electromagnetic signals normally incident upon a plasma gradient. An electron collision frequency that is independent of position in the plasma interface is assumed. A "slide rule" is provided to enable quick determination of the magnitude of the reflection coefficient for "very overdense, exponential plasmas" in the lower atmosphere (to 70 km) for frequencies from 30 MHz to 10,000 MHz. For plasmas that only reach a finite density, a complete set of graphs is given presenting power-reflection coefficients for normalized plasma parameters. Reflectivities are plotted if greater than 10^{-20}. Normalized collision frequencies (Z) from 10^{-3} to 10^{+3} are covered. Normalized plasma to RF frequency ratios inside the plasma cover the range from 0.01 to 100 ($10^{-4} \leq X \leq 10^{+4}$). Characteristic distances over which the plasma density exponentiates in the gradient region vary from 10^{-3} wavelengths to 10^{+3} wavelengths. Simple aids to the reader are provided so that he may easily determine his parameters for use with the reflectivity curves, should his plasma be of atmospheric constituents.</p>			

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